SOLAR SOUND SPEED INFERRED FROM SoHO HELIOSEISMIC DATA

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ABSTRACT

Inversions of the GOLF frequencies complemented with the 144 days MDI data have been performed to obtain the solar sound speed and density with a regularized least square method. We discuss the choice of the inversion parameters and the sensitivity of the inferred sound speed and density relatively to these parameters.

Key words: Helioseismology

1. INTRODUCTION

Different methods for inverting the solar oscillations frequencies have been used to infer sound speed and density in the solar interior. The results obtained from the data provided by the various helioseismic experiments lead to the conclusion of an agreement between solar sound speed and the up to date solar model sound speed within 2.10^{-3}. However a discrepancy appearing just below the convection zone is probably due to the lack of some additional mixing in the model. This feature is well established because many solar oscillation modes are observed which probe this region of the Sun. On the contrary, the structure of the solar core is not yet precisely obtained because few modes penetrate in this region and their frequencies are not easy to determine with great accuracy. It is the aim of the whole solar disc observations to obtain these frequencies. Different results have been derived for the solar core structure when considering the various sets of data provided by these observations. Inversions of solar sound speed with GOLF and LOWL data have been presented in Turck-Chièze et al. (1997) using SOLA method. Here we consider the GOLF frequencies with degrees $l = 0, 1, 2, 3$ obtained from one year of data beginning on April 11th 1996 (Lazrek et al. (1998)) and the MDI frequencies (Rhodes et al. (1997)) to infer the solar sound speed using a regularized least square method (RLS) and we discuss the choice of the inversion parameters and the sensitivity of the solution to the smoothing constraints.

2. THE RLS INVERSION METHOD

For each mode $i \equiv \{n, l\}$, $i = 1, N$, the relative differences $\delta\nu_i/\nu_i$ between solar frequencies and the frequencies ν_i of a solar model are expressed as:

$$\frac{\delta\nu_i}{\nu_i} = \int_0^{R_\odot} K^i_{c_0} \frac{\delta c}{c_0} \mathrm{d}r + \int_0^{R_\odot} K^i_{\rho_0} \frac{\delta\rho}{\rho_0} \mathrm{d}r + \frac{F(\nu_i)}{Q_i} + \epsilon_i \quad (1)$$

Here ϵ_i is the observational error, $F(\nu_i)/Q_i$ represents the contribution of the uncertainties on physics and structure of surface layers, $Q_i = E_i(\nu_i)/E_{l=0}(\nu_i)$ is a normalized energy of mode i, $\delta c/c_0$ and $\delta\rho/\rho_0$ are the relative differences in sound speed and density between the Sun and the model and $K^i_{c_0}$ and $K^i_{\rho_0}$ are the related kernels.

The two independent unknown functions $\delta c/c_0$ and $\delta\rho/\rho_0$ and the surface term $F(\nu_i)$ are developed respectively on piece-wise polynomials of second order in r and Legendre polynomials of ν:

$$\frac{\delta c}{c_0} = \sum_{j=1}^{N_f} c^1_j \psi_j \quad ; \quad \frac{\delta\rho}{\rho_0} = \sum_{j=1}^{N_f} c^2_j \psi_j \quad ; \quad (2)$$

$$\frac{F(\nu_i)}{Q_i} = \sum_{p=1}^{N_s} \beta_p \phi_p(\nu_i) \quad (3)$$

The coefficients $\mathbf{C} = \{c^1_j, c^2_j, \beta_p\}_{\substack{j=1,N_f \\ p=1,N_s}}$ are obtained by minimizing the quantity:

$$\mathbf{J(C)} = (N - N_s - 2N_f)\chi^2_\nu + \mu_c T_r = \chi^2 + \mu_c T_r \quad (4)$$

with $\chi^2 = \sum_1^N r_i^2$

$$r_i = \frac{\frac{\delta\nu_i}{\nu_i} - \left[\int_0^{R_\odot} K^i_{c_0} \frac{\delta c}{c_0} \mathrm{d}r + \int_0^{R_\odot} K^i_{\rho_0} \frac{\delta\rho}{\rho_0} \mathrm{d}r + \frac{F(\nu_i)}{Q_i}\right]}{\sigma_i} \quad (5)$$

and a regularizing term:

$$T_r = \int_0^{R_\odot} f(r) \left[\left(\frac{\mathrm{d}^k}{\mathrm{d}r^k} \frac{\delta c}{c_0}\right)^2 + \frac{\mu_\rho}{\mu_c} \left(\frac{\mathrm{d}^k}{\mathrm{d}r^k} \frac{\delta\rho}{\rho_0}\right)^2\right] \mathrm{d}r \quad (6)$$

N is the number of observed frequencies and σ_i^2 the variance of observational errors assumed to be gaussian. We have taken $f(r) = 1$ for $k = 1$ and $f(r) = r^{-2}$ for $k = 2$. The solution of the inversion depends crucially on the choice of the different parameters which are to be fixed: the number N_s of components in the surface term development, the trade-off parameter μ_c, the ratio μ_ρ/μ_c, the number N_f of the grid points and their distribution, here taken according to the density of turning points along the radius.

The number of grid points and the number of surface terms have been determined by looking at the condition number of the system of Euler equations derived from the minimization of $\mathbf{J}(\mathbf{C})$ (ratio of larger to smaller singular value of the system) and at the residuals r_i which should be randomly distributed as a function of the frequency and turning points (Basu & Thompson (1996)). This property is tested by computing the number N_{ri} of sign changes of the residuals when they are ordered in increasing frequencies or turning points radius and comparing this number to the expected value $N/2 \pm \sqrt{N}/2$ when the distribution is random. We introduce a factor $q = (2N_{ri} - N)/\sqrt{(N)}$, q_ν (when ordered in frequencies) and q_{rt} (when ordered in increasing turning points positions) which should be less than 1.

The trade-off parameter μ_c is determined by plotting the so-called L-curve which gives the smoothing term $\log(T_r)$ as a function of χ^2_ν at given μ_ρ/μ_c ratio. We have to find a compromise between a good fit of the data (small χ^2_ν) and a rather smooth physically acceptable solution (small regularization term $\log(T_r)$).

Two smoothing constraints, first and second derivative ($k = 1, 2$ in Equation 6) of the sound speed and density relative differences have been tested.

3. INVERSION OF THE SOLAR STRUCTURE

3.1. Data and choice of the parameters

We have used the 144 days MDI data (Rhodes et al. (1997)) for degrees $l > 3$ and the GOLF frequencies for degrees $l \leq 3$ (Lazrek et al. (1998)). In Figure 1, we have plotted the relative differences between the observed frequencies and those of the standard solar model S of Christensen-Dalsgaard et al. (1996) normalized by the quantity Q_i with the observational error bars.

The relative frequency differences of the f-modes detected by MDI have a nearly constant positive value (about 0.2×10^{-3}) for all frequencies. This can be interpreted as a shift $-3Q_i\delta R/2R$ due to a solar radius smaller than the solar model. With $Q_i \sim 0.3$ for the f-modes, the corresponding difference in radius is of the order of 300 km, in agreement with the results obtained by Antia (1997), Schou et al. (1997), Brown & Christensen-Dalsgaard (1998). These modes have been excluded in the inversion.

According to the previously defined criteria, we chose a moderate number of surface terms $N_s = 12$. For the first derivative smoothing constraint, L-curve and q_ν are plotted for different values of the trade-off parameter μ_c and different values of the number of grid points in Figure 2.

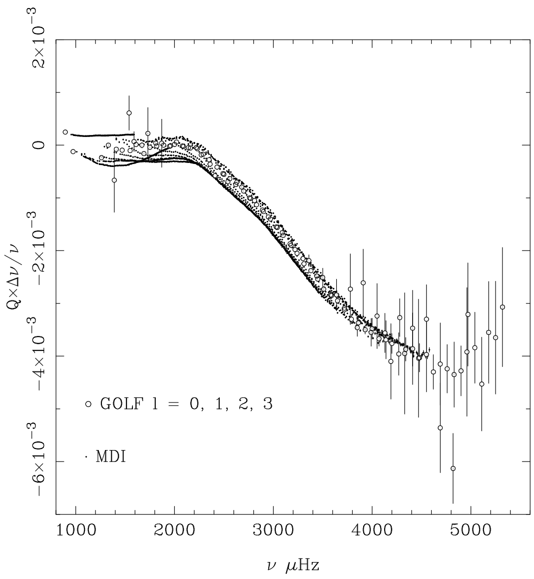

Figure 1. Normalized relative frequency differences between observations (GOLF + MDI) and a solar model.

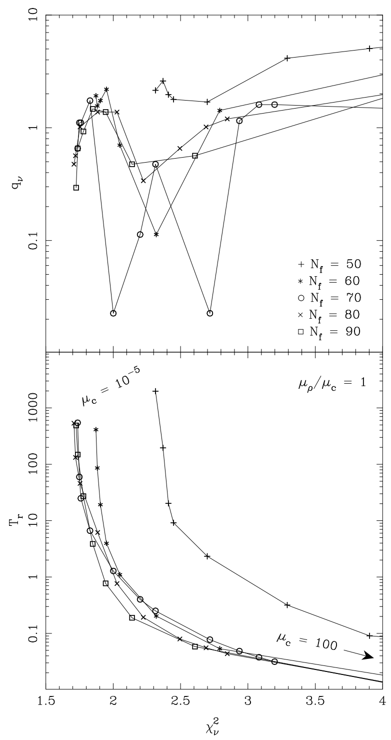

Figure 2. Upper panel: variation of the factor q_ν characterizing the randomness of the distribution of the residuals r_i for different values of the trade-off parameter. Each curve corresponds to a given number of grid points $N_f = 50$ to 90. Lower panel: L-curves for different number of grid points.

It is seen that a reasonable choice of N_f is 70 to 80.

Below that value, χ^2_ν is too large and for $N_f = 50$ the factor q_ν is always larger than 1.

The ratio μ_ρ/μ_c allows to adjust the smoothing of the density relatively to the sound speed. Figure 3 shows the L-curves and the factors q_ν and q_{rt} for two values of μ_ρ/μ_c. It appears that q_{rt} is always smaller than unity which means that the distribution of the residuals ordered in turning points is random. It is not the case for q_ν. We obtain lower values of χ^2_ν and q_ν with μ_ρ/μ_c around 0.25. This ratio value will be used in the following.

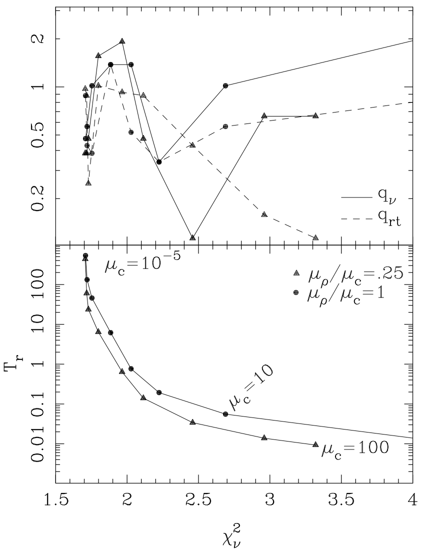

Figure 3. L-curves (lower panel) and the q_ν and q_{rt} factors (upper panel) for two values of μ_ρ/μ_c and $N_f = 80$.

3.2. Results for sound speed and density

The relative differences between solar and theoretical sound speed and density are plotted as a function of the radius in Figure 4. The errors on the solution, derived from the errors on the coefficients **C** are plotted in dashed lines. The results are in agreement with those obtained by Turck-Chièze et al. (1997) with the SOLA method. According to Figure 2, the minimum value of χ^2_ν is 1.7, but in order to obtain a solution reasonably smooth, a rather large value of the trade-off parameter μ_c has been used, corresponding to a χ^2_ν significantly larger than unity. Here $\chi^2_\nu = 2.8$. About 10 modes have large residuals. If they are omitted in the inversion, χ^2_ν is lowered by 0.2. The large χ^2_ν value may come from different sources, systematic or underestimated errors in the observational data, an imperfect removing of the effect of near-surface uncertainties in the physics through the function $F(\nu)$. It can be noted that, while the sound speed difference is rather stable when the trade-off parameters are slightly modified, the density difference is much more sensitive.

It may be instructive to consider the average contributions to the total χ^2 of modes with given degree $S_l = \sum_n r_i^2/N_l$ and of modes with given radial order

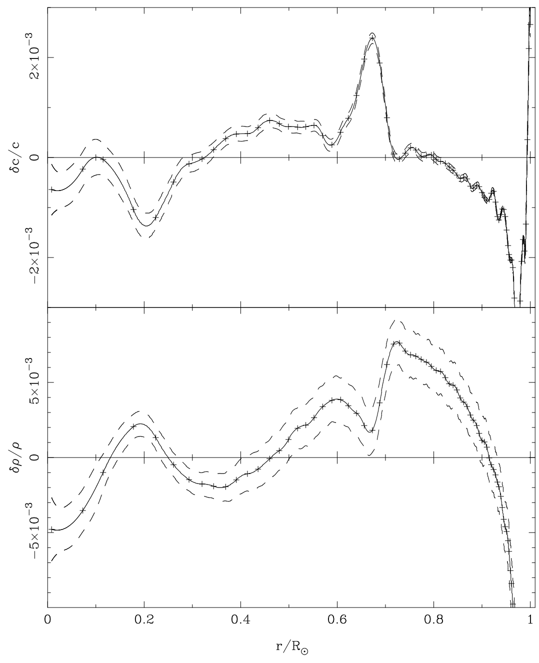

Figure 4. Relative difference in sound speed and density between the Sun and the solar model S with first derivative smoothing.

$S_n = \sum_l r_i^2/N_n$ in order to detect systematic effects (N_l and N_n are respectively the number of modes of degree l and order n). Figure 5 gives the values of S_l and S_n (modes $l = 108$, $n = 8$ and $l = 145$, $n = 6$ have been omitted due to their large residuals). The average contributions of the modes of given l are of the same order for all l except in the domain $l = 166$ to $l = 171$ and for $l > 190$ where only $n = 1$ is present. The average contributions of the modes of given n decrease globally when n increases for $n < 30$.

The sound speed difference that we obtain corresponds to a global fit with the observational data. Due to the fact that the coefficients **C** are linear combinations of the data, the inferred sound speed difference $\delta\overline{c}(r_0)/c_0$ at one point r_0 can be expressed as a sum of an average of the real one $\delta c(r_0)/c_0$ with averaging kernel $\mathbf{K}_c(r, r_0)$ and contributions of the density through a kernel $\mathbf{K}_\rho(r, r_0)$ and of the surface which are assumed to be small.

$$\frac{\delta\overline{c}(r_0)}{c_0} = \int_0^{R_\odot} \mathbf{K}_c(r, r_0)\frac{\delta c}{c_0}\mathrm{d}r + \int_0^{R_\odot} \mathbf{K}_\rho(r, r_0)\frac{\delta\rho}{\rho_0}\mathrm{d}r + T_s \quad (7)$$

The kernels $\mathbf{K}_c(r, r_0)$ and $\mathbf{K}_\rho(r, r_0)$ are plotted in Figure 6 for $r_0 = 0.2$ and $r_0 = 0.7$. As expected $\mathbf{K}_c(r, r_0)$ is well localized while $\mathbf{K}_\rho(r, r_0)$ is small and oscillating. However they have both some amplitude at the surface. This amplitude is small for $r_0 = 0.7$ but larger for $r_0 = 0.2$. The surface term T_s is always found negligible.

3.3. Sensitivity to the smoothing parameter

We have performed the same work with regularizing term involving the second derivative. We use the

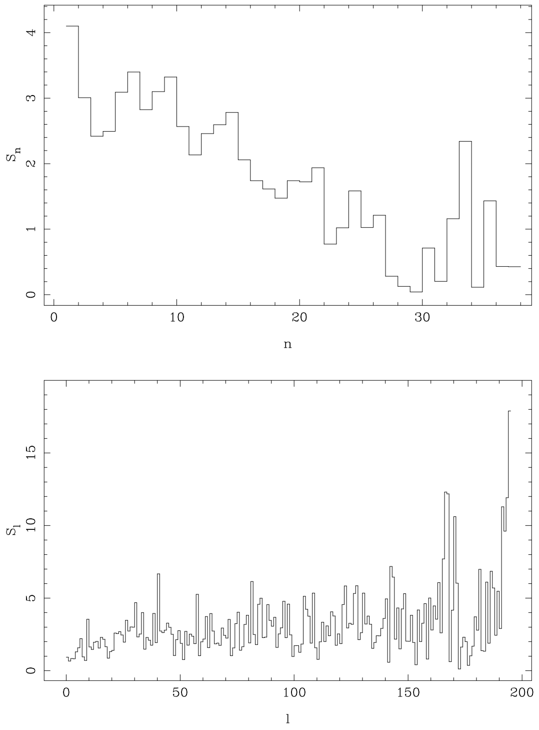

Figure 5. Average contributions to the total χ^2 of modes with given degree S_l (lower panel) and of modes with given radial order S_n (upper panel).

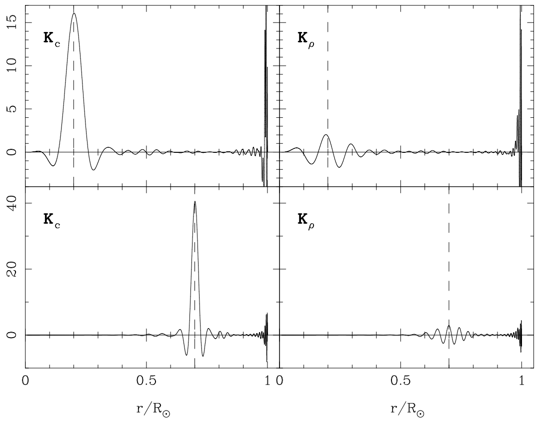

Figure 6. Averaging kernels for the sound speed RLS inversion at two points $r_0 = 0.2$ and 0.7.

same number of surface terms and grid points. The trade-off parameters are chosen in the same way as before. The relative difference in sound speed and density between the Sun and a standard solar model are given in Figure 7 for the two smoothing constraints. Concerning the sound speed, the results of the two inversions agree very well below $r \sim 0.85$ down to $r \sim 0.15$. In the core where the data do not constrain strongly the solution, they differ noticeably but within the limit of the errors. It results that the sound speed of the solar core derived with the GOLF low degree modes is smaller than that of the standard model we have used, in agreement with the results of

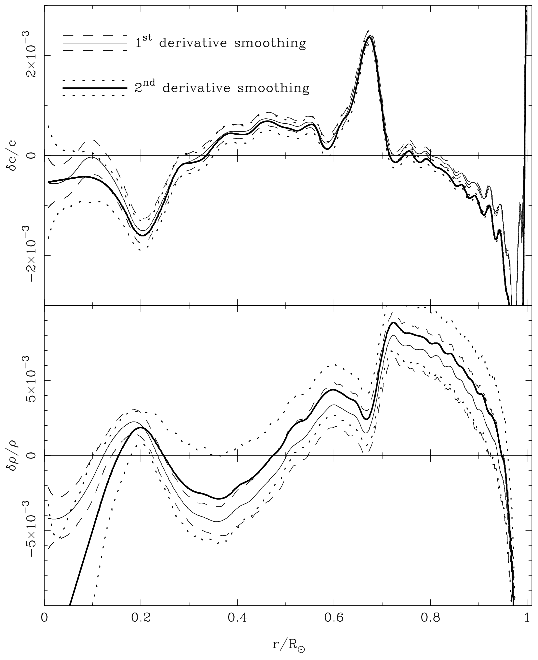

Figure 7. Relative difference in sound speed and density between the Sun and the model obtained by RLS inversion with first and second derivative smoothing constraint.

Turck-Chièze et al. (1997). There is also a discrepancy between the two solutions in the upper layers. Here we have not used the f-modes in the inversion. If they are added, χ^2 is much increased but the solution is slightly modified in the center. Inversions from a solar model with a smaller radius should be investigated.

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