LOWL P-MODE FREQUENCIES AND THEIR VARIATION WITH SOLAR ACTIVITY

Sebastián J. Jiménez-Reyes[1,2]**, Thierry Corbard**[1]**, Pere L. Pallé**[2]**, and Steve Tomczyk**[1]

[1]High Altitude Observatory, NCAR, PO Box 3000, Boulder, CO 80307 USA

[2]Instituto de Astrofísica de Canarias, E-38701, La Laguna, Tenerife, Spain

ABSTRACT

We present an analysis of the frequency shift and the even terms of the frequency splitting coefficients carried out using six years of LOWL data, starting in 1994. The temporal variations, and their dependences with the frequency and degree are addressed. The results are consistent with previous analysis.

1. INTRODUCTION

It is very well established that the variations of some of the oscillation mode parameters is a signature of the solar activity cycle. For instance, the acoustic mode frequency is shifted positively of 0.4μHz from minimum to maximum of the solar cycle (Libbrecht and Woodard 1990, Régulo et al. 1994, Jiménez-Reyes et al. 1998). It has also been noted that, this variation depends on the frequency and on the degree of the mode as well. The even terms of the splitting coefficients present similar changes. Here, we present an analysis of the frequency shift using LOWL observations.

2. DATA ANALYSIS AND RESULTS

The raw data consist in full disk solar Doppler images, which have been collected by LOWL instrument. This experiment is based on a Magneto-Optical Filter and has demonstrated its high sensitivity to the solar oscillations. The data were analyzed using the LOWL pipeline which has recently been improved (Jiménez-Reyes 2000). The images are first calibrated and then a spherical harmonic decomposition is performed in order to create time series for degrees from ℓ=0 to 99. Finally, a Fast Fourier Transform is applied to the time series.

The spherical harmonics are not orthogonal over the observed area, limited to one hemisphere. Therefore, the modes cannot be totally isolated and some correlations between different (ℓ, m) spectra exist. Historically, the statistic of the power spectra has been assumed to be as χ^2 with 2 degrees of freedom. However, this assumption is not correct when the spectra are not independent. Recently, important improvements have been achieved in the fitting techniques to be applied to observations made using instruments with spatial resolution (Schou 1992, Appourchaux et al. 1998). The real and the imaginary part of the Fourier Transform follow a multi-normal distribution which is described by a covariance matrix. The likelihood function under these assumptions can be written as:

$$S(\vec{a}) = \sum_{i=1}^{N} \log \mid E(\vec{a},\nu_i) \mid + y^T(\nu_i)E(\vec{a},\nu_i)y(\nu_i) \quad (1)$$

where $y(\nu_i)$ corresponds to the Fourier Transform and $E(\vec{a},\nu_i)$ is the covariance matrix calculated by:

$$E_{nm}(\vec{a},\nu_i) = \sum_{m'=-\ell}^{\ell} C_{nm'}C_{mm'}v_{m'}(\vec{a},\nu_i) + B_{nm} \quad (2)$$

where C is the leakage matrix which represents the correlation between modes with different m (m-leakage), whereas B is the noise covariance matrix which gives information about the noise correlation between spectra. $v(\vec{a},\nu)$ is the variance and it is given by a simple Lorentzian profile defined by the parameters $\vec{a}$ that we are trying to infer using a maximum likelihood method. The shift between m-components is given by:

$$\nu_{n\ell m} = \nu_{n\ell} + \sum_{i=1}^{n_{coef}} a_i(n,\ell)P_i^{\ell}(m) \quad (3)$$

where $P_i^{\ell}(m)$ are orthogonal polynomials normalized such that $P_i^{\ell}(\ell) = \ell$ (Schou et al. 1994, App. A). The odd a-coefficients are induced by the internal rotation while the even terms are mainly related to effects of second order due to rotation, the presence of magnetic fields or any departure from spherical symmetry.

The distance between modes with the same order n and with $\Delta\ell$=± 1 get closer and closer at high degrees leading to a leakage of the modes (ℓ-leakage). This problem arises at lower degree in the case of observations from just one station, due to the existence of the sidelobes as a result of the modulation of one day in the signal. That is particular important for those modes where

$\nu/L \approx 31\mu$Hz. In order to reduce the systematic errors an extra covariance matrix is added to take into account the presence of spurious modes which can be close to the target mode. Nevertheless, we have seen that the systematic errors are specially important in the odd a-coefficients and not in the even ones.

At high degree, the numerical evaluation of the likelihood function get slow, mainly because of the large dimensions of the covariance matrix. Therefore, we use just the main diagonal of the covariance matrix. We have checked that this is indeed a very good approximation for high degrees.

3. FREQUENCY SHIFT

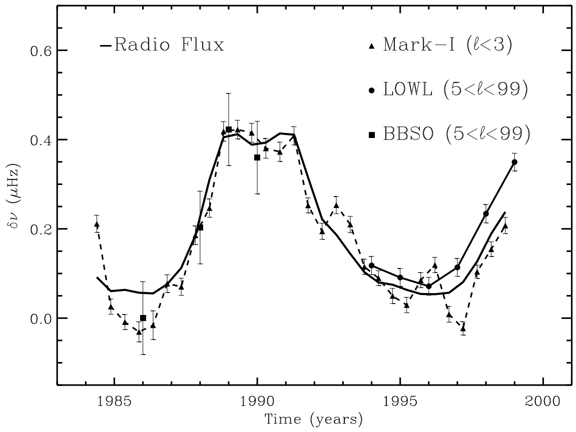

Figure 1. Time variation of the integrated frequency shift. The BBSO data have been averaged in the same way and plotted in the figure. In addition, 15 years of Mark-I frequency shifts are shown as well.

Figure 1 shows the temporal variation of the integrated frequency shift for frequencies between 2 and 3.5 mHz. For comparison, the figure includes the same calculations using the BBSO data (squares). Mark-I instrument has been measuring the solar oscillation for more than 20 years. This instrument observes the Sun like a star, so the information is limited to low degrees ($\ell \leq 3$). Recently, the database has been re-analyzed (Jiménez-Reyes 2000) over a much wider time interval than before. Here, we have calculated the yearly frequency shift every six months for the last 15 years . All the results are very well correlated with the changes in the solar activity cycle denoted on Fig. 1 by the best linear fit between the radio flux and the results from Mark-I. The amplitude of these changes are, for low degrees, close to 0.4μHz, peak-to-peak, and the correlation with the radio flux variations is close to 1.

The temporal variations of the central frequency are expected to change with both frequency and degree. Earlier works (Libbrecht and Woodard, 1990) have shown a strong variation with the frequency, the frequency shift being null at $\sim$2 mHz and increasing at higher frequency. The ℓ-dependence has been found to be more weak.

Figure 2 shows the variation of the frequency shift averaged over ℓ for three selected years. The reference is

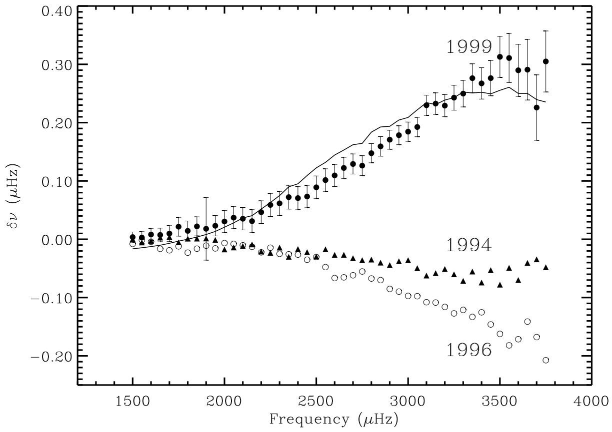

Figure 2. Frequency dependence of the frequency shift for three years. The error bars have been shown just for one year for clarity. The solid line represents the best linear fit of the inverse mode mass to the 1999 observations.

the mean over the six years of data. The last minimum of the solar cycle happened in 1996, whereas 1999 represents the year with higher level of solar activity in our database. As we can see, the frequency dependence is very clear, mainly between 1996 and 1999. The inverse

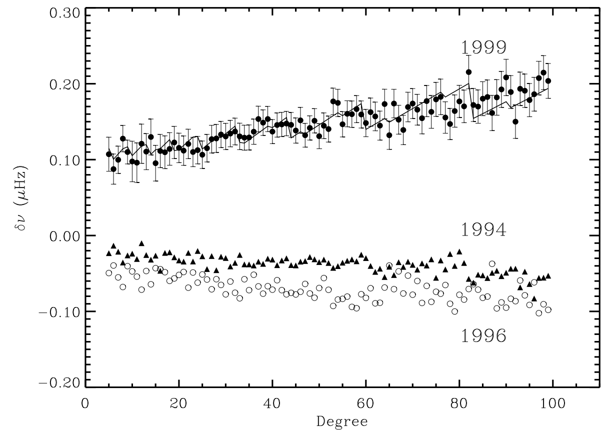

Figure 3. ℓ-dependence of the frequency shifts for three different years. The solid line through the results corresponding to 1999, denotes the best linear fit to the inverse mode mass.

mode mass has been calculated using the model of Morel et al. (1997). It has been averaged in the same way as the results shown in Figure 2. The solid line over the points corresponding to 1999 represents the best linear fit to inverse mode mass. The observed frequency shift exhibits almost the same frequency dependence than the inverse mode mass. However, there are still some differences which can be seen also in the analysis of the GONG data (Howe et al. 1999). This may or not be significant but it is interesting to notice that both results present the same fluctuations around the inverse mode mass curve.

The ℓ-dependence has been also calculated and plotted in Figure 3. Again, we have selected the same years than in the last figure and the reference is the mean over the six years. The dependence with the degree is maximum for 1999 with a difference of about 0.1μHz between ℓ=5 and ℓ=99. The inverse mode mass, averaged in the same way than the observations, is shown in the figure, scaled to the best linear fit using the results corresponding to 1999. The solid line follows remarkably well the frequency shift.

The variations of the frequency shift have been studied, assuming a linear relationship with the solar activity indices.

The solar indices used here are:

- the sunspot number,R_I;
- the integrated radio flux at 10.7cm, F_{10} (both obtained from the Solar Geophysical Data);
- the Kitt Peak magnetic index (KPMI) extracted from the Kitt Peak full disk magnetograms (Harvey 1984);
- the Mount Wilson index also called Magnetic Plage Strength Index, MPSI (Ulrich et al. 1991);
- the equivalent width of HeI 10830Å averaged over the whole solar disk using data from Kitt Peak observatory;
- the Total Solar Irradiance, TSI (Fröhlich, C. & Lean, J., 1998).

The results shown in Tab. 1 can be compared with those obtained recently by Howe et al. (1999) and Bhatnagar et al. (1999) from GONG observations. The magnitude of these parameters are very similar although some differences remain. The reason could be the different range in frequency taken to calculate the average. While we are taking frequencies between 2 and 3.5 mHz, Bhatnagar et al. (1999) were taking all the result between 1500 and 3500μHz and Howe et al. (1999) were using a mean calculated at 3mHz.

Table 1. Results of the weighted linear least-squares fits for the frequency shifts as a function of different solar indices.

Index	Intercept (μHz)	Slope
R_I	-0.0314±0.0070	0.0031±0.0001[a]
F_{10}	-0.2245±0.0131	0.0032±0.0001[b]
KPMI	-0.2203±0.0191	0.0317±0.0018[c]
MPSI	-0.0168±0.0063	0.1604±0.0068[d]
He	-0.5286±0.0416	0.0114±0.0008[e]
TSI	-363.4905±52.6823	0.2662±0.0386[f]

a:μHz; b:μHz/(10^{-22} J/s/m^2/H); c,d:μHz G^{-1}; e:μHz mÅ^{-1}; f:μHz W^{-1}m^2

4. THE EVEN A-COEFFICIENTS

The even terms are expected to change with the frequency. Therefore, they were ℓ-averaged over short regions in frequency, as we did with the frequency shift. The results for two selected years, (1996 in the bottom and 1999 in the top) are shown in Figure 4. The even a-coefficients are close to zero in 1996, when the solar cycle reached the last minimum. However, a big change can be seen in 1999, following again the same frequency dependence than the inverse mode mass. Again, there is a clear fluctuation of a_2 around the inverse mode mass curve. Nevertheless this may not be significant as it stays within the error bars.

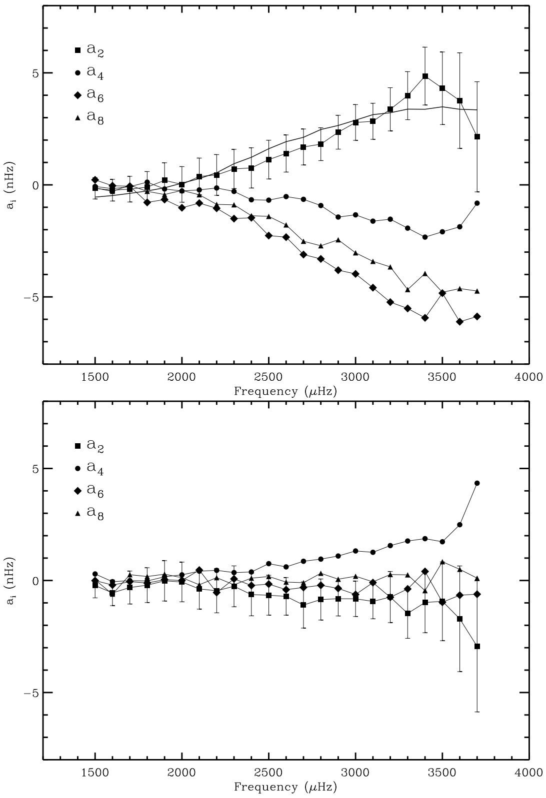

Figure 4. Frequency dependence of the even a-coefficients for two different years, 1996 (bottom) and 1999 (top).

The time-variation of the solar cycle does not affect only the central frequency. The changes in the even a-coefficients demonstrate that, whatever is the perturbation leading to these variations, it depends on the latitude.

We have integrated the even terms for each one of the time series. The resulting values are shown in Figure 5 against the radio flux calculated over the same period. The straight line represents the best linear fit of both quantities.

5. CONCLUSION

We have analyzed six years of LOWL data from 1994 to the end of 1999 in order to parameterize the variation of the frequency shift and the even a-coefficients.

In summary, the acoustic central frequency presents a variation which is very well correlated with the solar cycle. The integrated amplitude of this variation, peak-to-peak, is close to 0.4μHz. This variation is however a

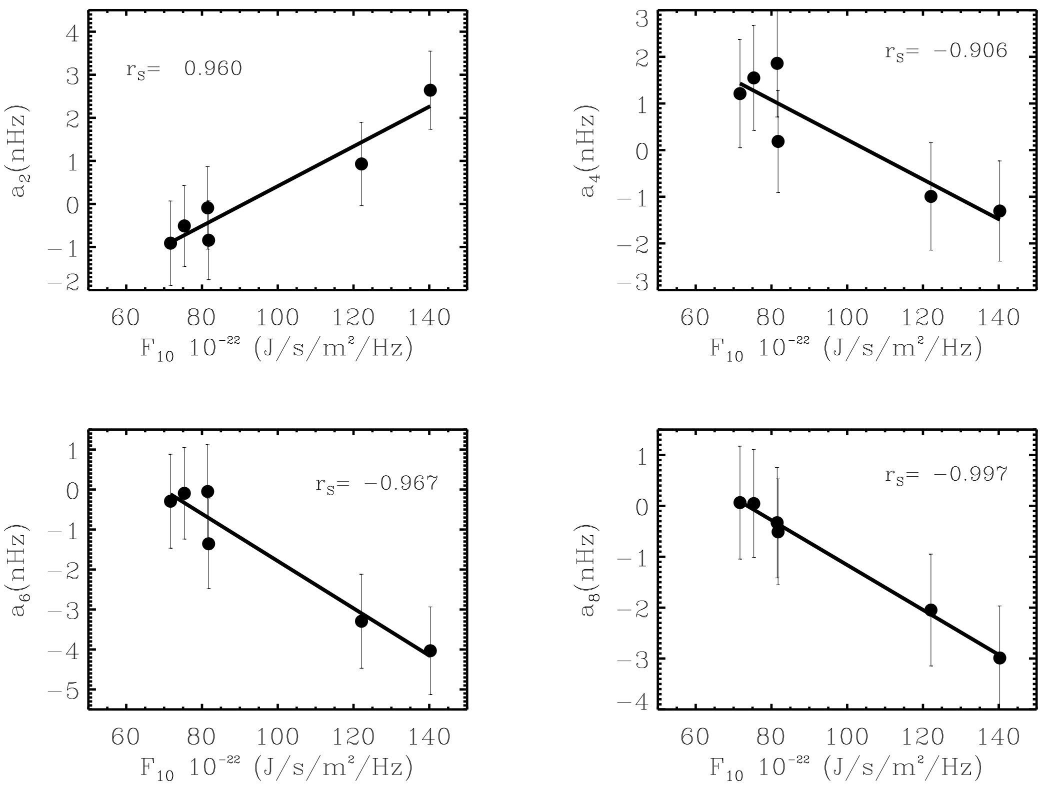

Figure 5. Variation of the integrated even a-coefficients against the radio flux at 10.7.cm. The solid line shows the best linear fit.

function of both the frequency and the degree, and these dependences can be well fitted by the inverse mode mass.

In addition, the even a-coefficients present a significant change with the solar cycle, very well correlated with the solar indices. Moreover, as for the frequency shift, these changes increase with the frequency. The results agree very well with earlier studies (Libbrecht and Woodard, 1990) carried out at the beginning of the last solar cycle.

LOWL instrument has been recently updated and a new instrument has been developed and installed at the Observatorio del Teide (Tenerife), leading to a new network called Experiment for Coordinated Helioseismic Observations (ECHO). ECHO is intended to continue observing for a complete solar cycle allowing us to better track the origin of the solar activity cycle.

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