The Solar Tachocline

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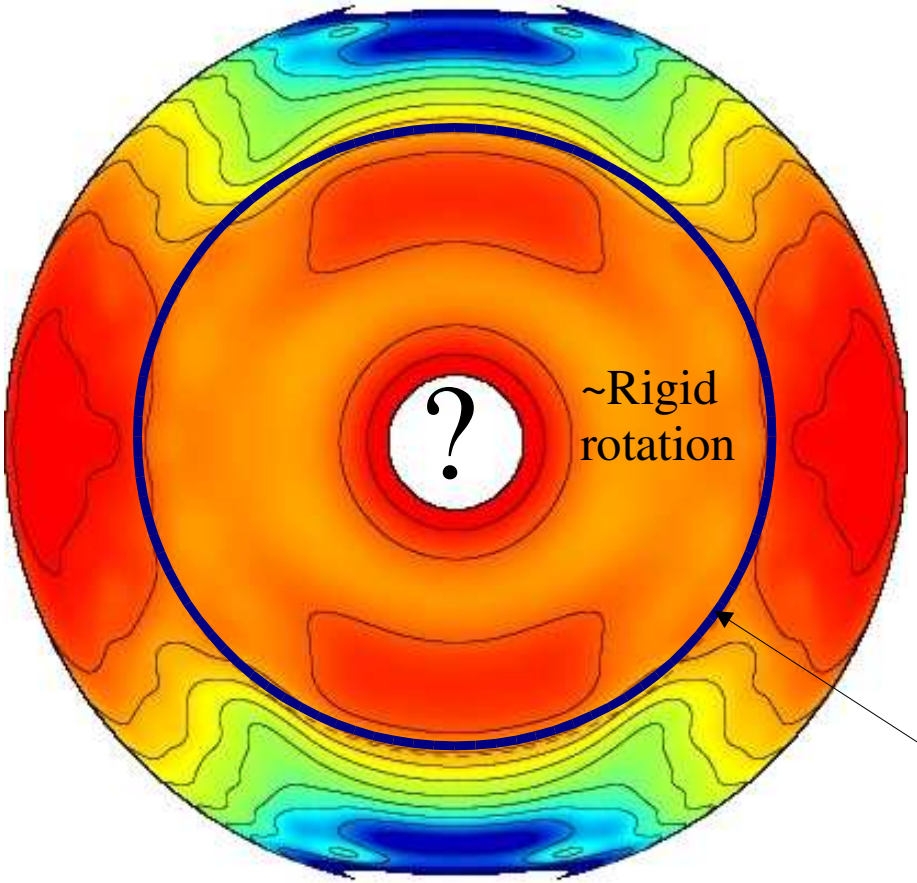
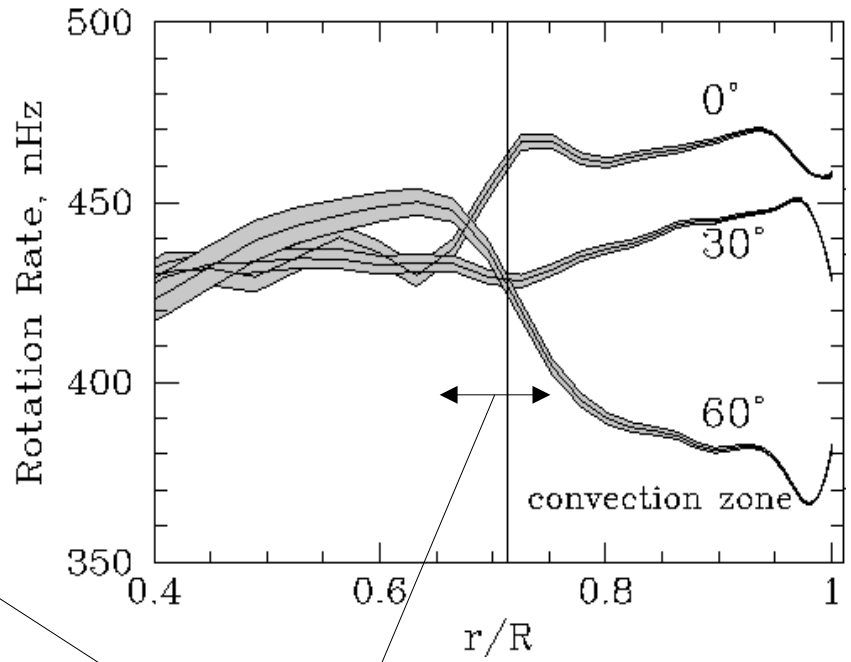
- Introduction: why so much interest?
- Methods for inferring the tachocline properties
- Observations: results and controversy
- Theories: A brief overview of recent developments
- Tachocline / overshoot and the mixing of chemical elements
- Tachocline / solar dynamo
- Conclusion and perspectives

Internal rotation rate from MDI data



460 Rotation Rate (nHz) 340

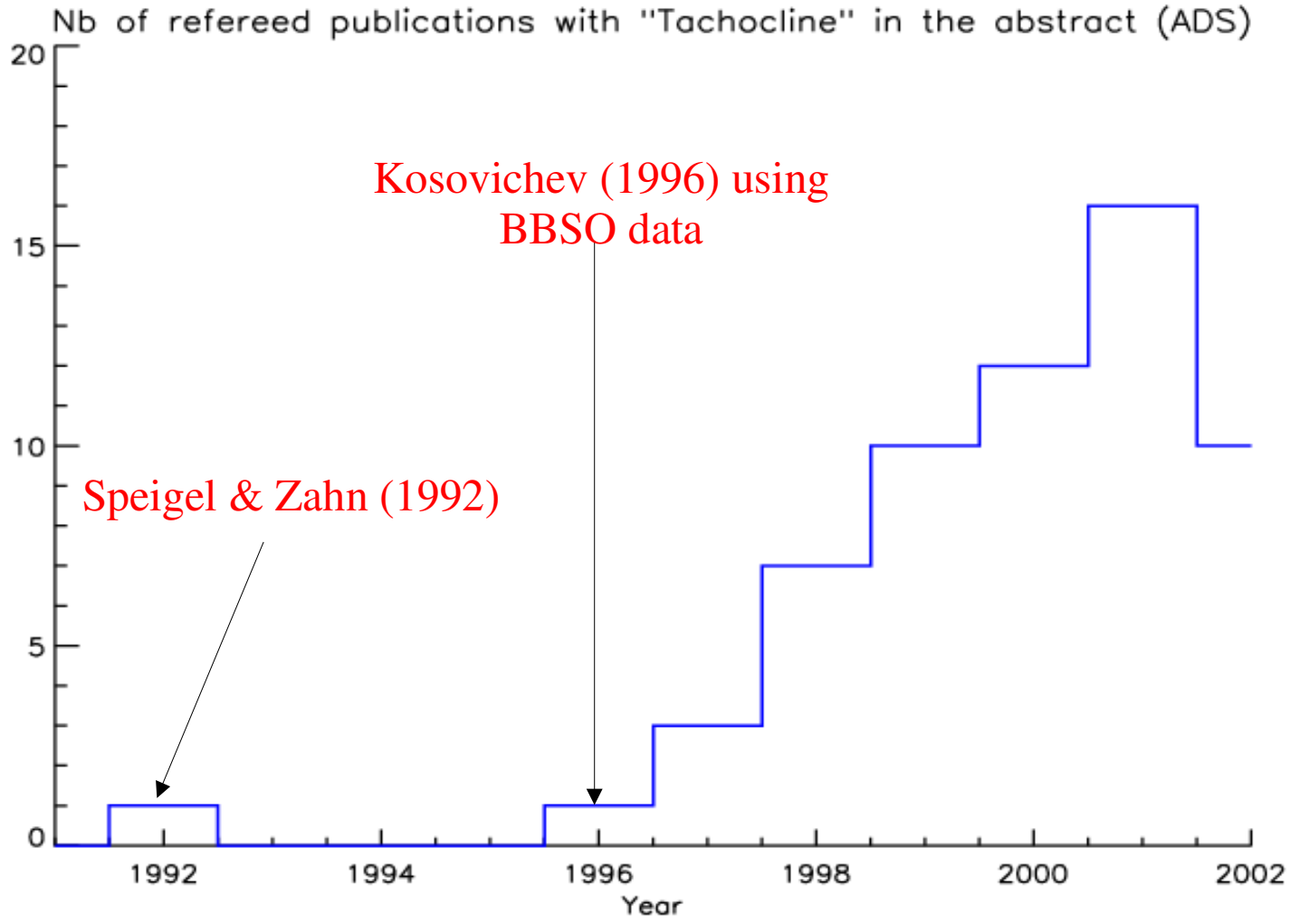
Corbard 1998



Kosovichev et al. 97

Tachocline
=
transition zone near the base of the Convection Zone where important radial shear occur

1992-2002: 10 years of Tachocline



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Why so much interest for the tachocline?

The tachocline has 2 basic properties the combination of which is unique in the solar interior:

1) Strong radial gradient of angular velocities

-> May induce shear turbulence

2)Location: close to the base of the convection zone (0.713 $R_\odot$)

-> Transition between the stably stratified radiative interior and the convectively unstable zone - overshoot layer ?

-> Possibility for the storage of a strong Magnetic Field.

-> Proximity of the Lithium burning zone (2.7 10^6K / 0.66 $R_\odot$)

-> Possibilities for the production of internal waves and transport of angular momentum

•Instabilities from Shear turbulence / MF / penetrative convection -> Possible mixing
+ Proximity of Lithium burning zone of chemical elements

•Shear + MF -> Ω-effect -> Best location for
3D Hydrodynamic or Magnetic instabilities -> α-effect the solar Dynamo

METHODS FOR INFERING

THE TACHOCLINE

PROPERTIES

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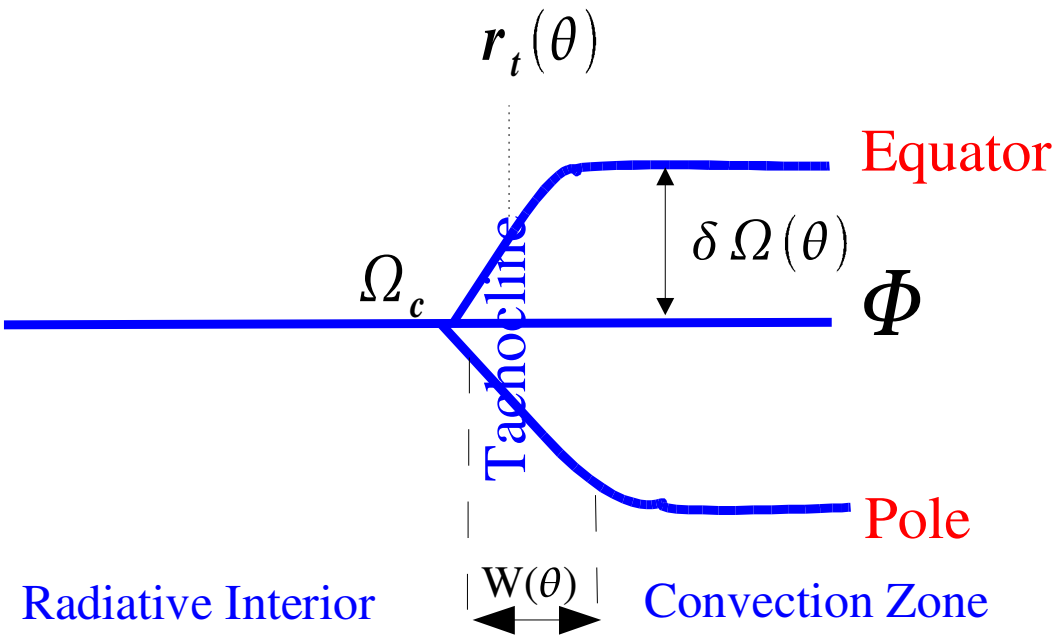
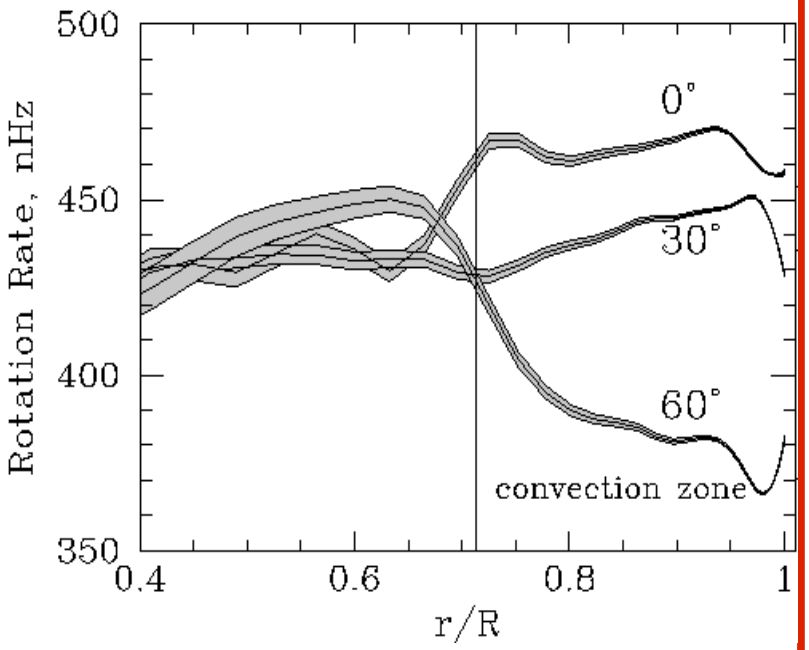
Definitions - Tachocline parameters





$$\Omega(r,\theta) = \Omega_c + \delta\Omega(\theta)\left(1 + erf\left(2\frac{(r - r_t(\theta))}{w(\theta)}\right)\right)$$

Charbonneau et al. (1997)

=> W(ϑ) corresponds to a change in the rotation rate of 85% of $\delta\Omega(\theta)$

Antia et al. (1998) use another form from which we can easily define the same quantities

Methods: Basic Equations

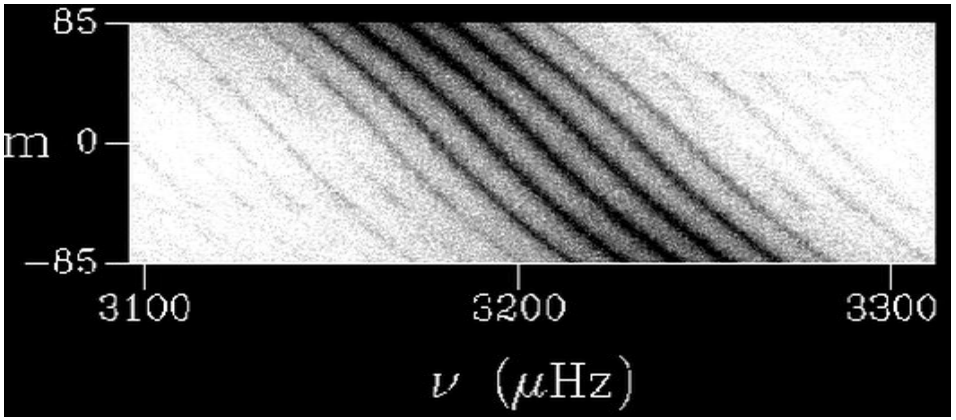
2D Integral equation: Frequency splittings <-> Internal Rotation rate

$$\Delta\nu_{nlm} = \frac{\nu_{nlm} - \nu_0}{m} = \iint K_{nlm}(r,\mu)\Omega(r,\mu)\,dr\,d\mu \qquad \mu = \cos(\theta)$$

Projection on polynomials

$$\nu_{nlm} - \nu_0 = \sum_{j=0}^{N_j} a_j P_j^l(m)$$

Odd a-coefficients:
Symmetrical part of the spectra
-> symmetrical part of the rotation / equator
Even a-coefficients:
centrifugal force / Magnetic field
any departure from spherical symmetry



l=85, n=8 *(GONG, Hill et al. 1996)*

1D Approximation in the equatorial plane (valid for $l >> 1$)

$$\Delta\nu_{nll} = \sum_{j=1,3,5..} a_j = \int K_{nl}(r)\Omega_1(r)\,dr$$

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Methods: linear inverse methods

Global Methods: Regularized Least Square (RLS) , Tikhonov

Compromise goodness of the fit <--> Smoothness

$$\chi^2(\Omega)=\frac{1}{N}\sum_{i=1}^{N}\left(\frac{\Delta\nu_i-\int K_i(r)\Omega(r)\,dr}{\sigma_i}\right)^2 \quad + \quad \lambda_r \qquad S(\Omega)=\int\left(\frac{\partial_p\Omega}{\partial r^p}\right)^2 dr$$

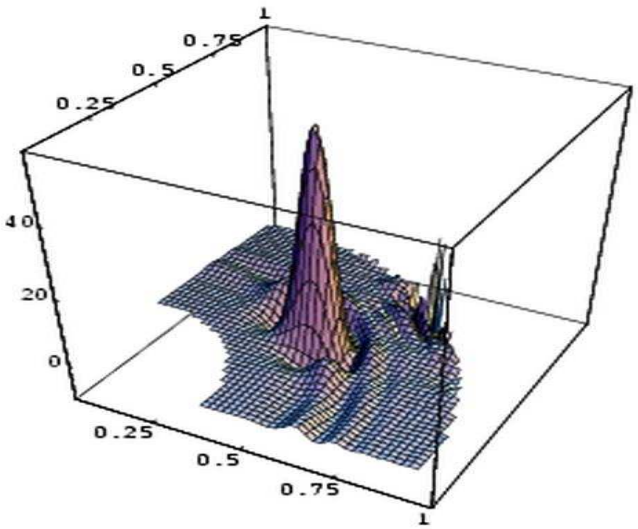
Local Methods: Optimal Localized Average (OLA,SOLA)

Compromise Resolution <--> Error Propagation

Linear method => $\Omega(r_0)=\sum_i c_i(r_0)\Delta\nu_i$

$$M\left(\sum_i c_i(r_0)K_i(r)\right) \quad + \quad \lambda \quad \sum_i c_i(r_0)\sigma_i^2$$

Averaging Kernels



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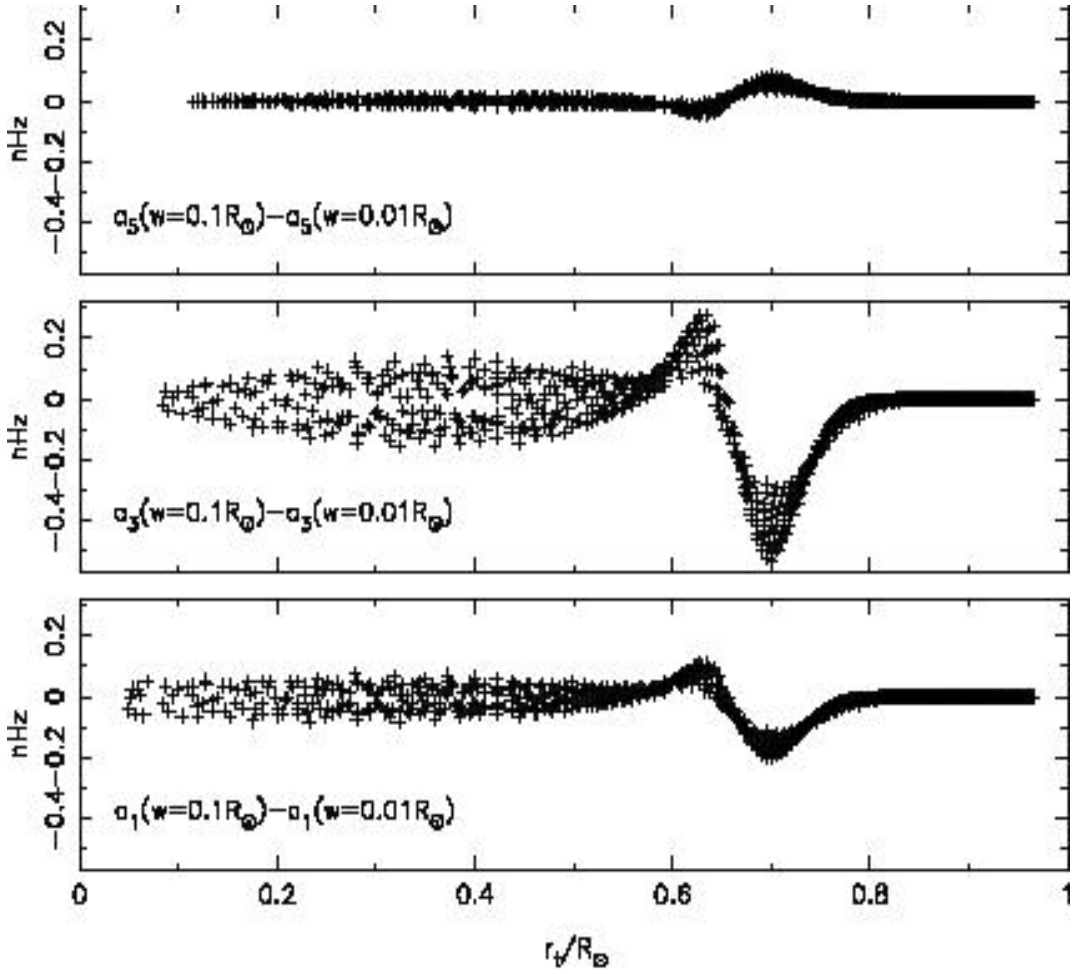
Methods: problems with the tachocline

Difficulties:

- **Amplitude of the signature of a change of width in the data**



Methods: problems with the tachocline

Difficulties:

- Resolution at the base of the convection zone ~ transition width
  => transition not resolved by linear methods
- Global methods (RLS) => global smoothness (everywhere)

3 **different approaches:**

(1) Direct Analysis: Model of the tachocline with few parameters

Simulated annealing *(Antia et al. 98)*, genetic algorithm *(Charbonneau et al., 98)*

(2) Local deconvolution or 'super resolution'

$$\tilde{\Omega}(r_0) = \int \kappa(r_0, r)\,\Omega(r)\,dr$$

$$\tilde{\Omega} = \kappa * \Omega$$

$$w_c = \sqrt{w^2 - 8\sigma^2}$$

Error function(w)

Gaussian (σ)

Error function(w_c)

(3) Non linear (adaptive) regularization

Iterative process which will adjust the regularization term based on the solution find at the previous step rather than on a global a-priori on the solution.

(Corbard et al. 99, Strakhov and Vorontsov 2001).

OBSERVATIONS:

RESULTS AND CONTROVERSY

Results (1): Equatorial plane

Data	$r_c/R_\odot$	$w/R_\odot$	Method	Ref
LOWL 94-96[a]	0.689 ± 0.006	0.01 ± 0.03	OLA + Deconvolution	Charbonneau et al. (1999)
LOWL 94-96[a]	0.691 ± 0.007	0.07 ± 0.03	RLS + Deconvolution	
LOWL 94-96[a]	0.695 ± 0.005	0.05 ± 0.03	RLS + Deconvolution	Corbard et al. (1998)
LOWL 94-96[a]	0.691 ± 0.004	0.01 ± 0.03	Adaptive Regularization	Corbard et al. (1999)
GONG 95-96[b]	0.685 ± 0.008	0.023 ± 0.041	FM (calibration)	Antia et al. (1998)
GONG 95-96[b]	0.684 ± 0.012	0.010 ± 0.010	FM (1D annealing)	
GONG 95-99[c]	0.689 ± 0.003	0.009 ± 0.010	FM (2D annealing)	Basu & Antia (2001)
MDI 96-00[d]	0.685 ± 0.004	0.019 ± 0.016	FM (2D annealing)	
GONG 95-99[c] + MDI 96-00[d]	0.689 ± 0.003	0.030 ± 0.006	FM	
mean and dispersion	0.689 ± 0.004	0.026 ± 0.022		

=> Low dispersion: very good agreement between all datasets and methods

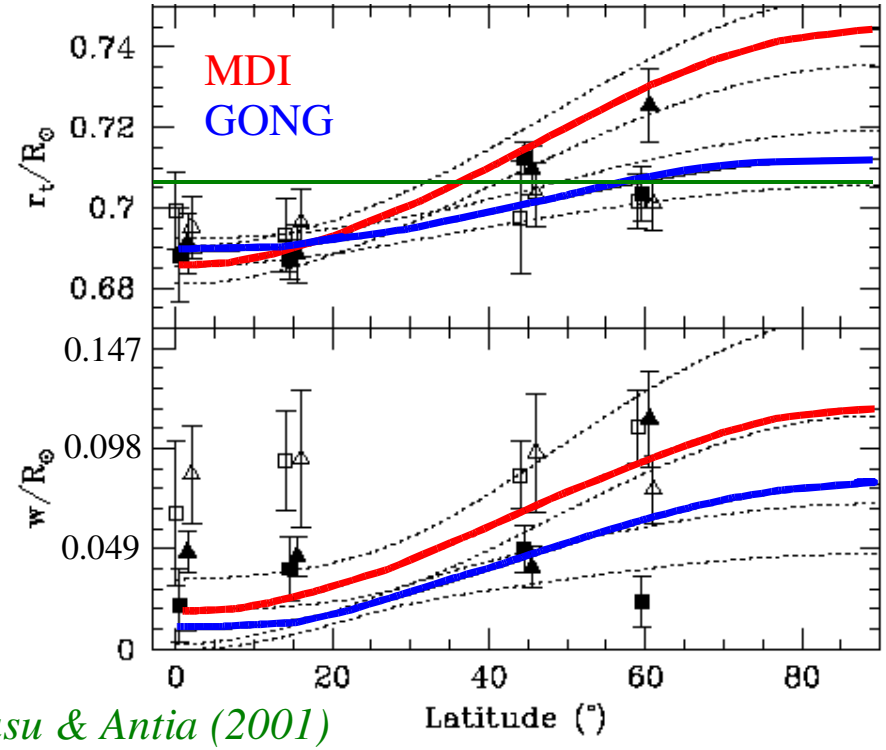
=> **The equatorial tachocline is centered significantly bellow the convection zone (~0.02$R_\odot$ bellow) and is no more the 0.05$R_\odot$ thick**

=> It is very likely that the equatorial tachocline lies completely beneath the convectively unstable zone in a thin subadiabatic layer.

Taking into account uncertainties in both position and width => **[0.66-0.72]$R_\odot$**

Results (2): Spherical average and latitudinal variations

Data	$r_c/R_\odot$	$w/R_\odot$	Method	Ref
BBSO 86,88-90	0.692 ± 0.005	0.09 ± 0.04	FM	Kosovichev (1996)
BBSO 86,88-90	0.707 ± 0.005	0.041 ± 0.026	FM (calibration)	Basu (1997)
GONG 95-96[e]	0.703 ± 0.006	0.049 ± 0.025	FM (calibration)	
GONG 95-96[f]	0.705 ± 0.004	0.051 ± 0.018	FM (calibration)	
LOWL 94-96[a]	0.705 ± 0.002	0.053 ± 0.015	Genetic FM	Charbonneau et al. (1999)
MDI 96-00[d]		0.042	Adaptive Regularization	Vorontsov et al. (2001)
mean and dispersion	0.702 ± 0.007	0.054 ± 0.018		



Basu & Antia (2001)

=> significant prolateness

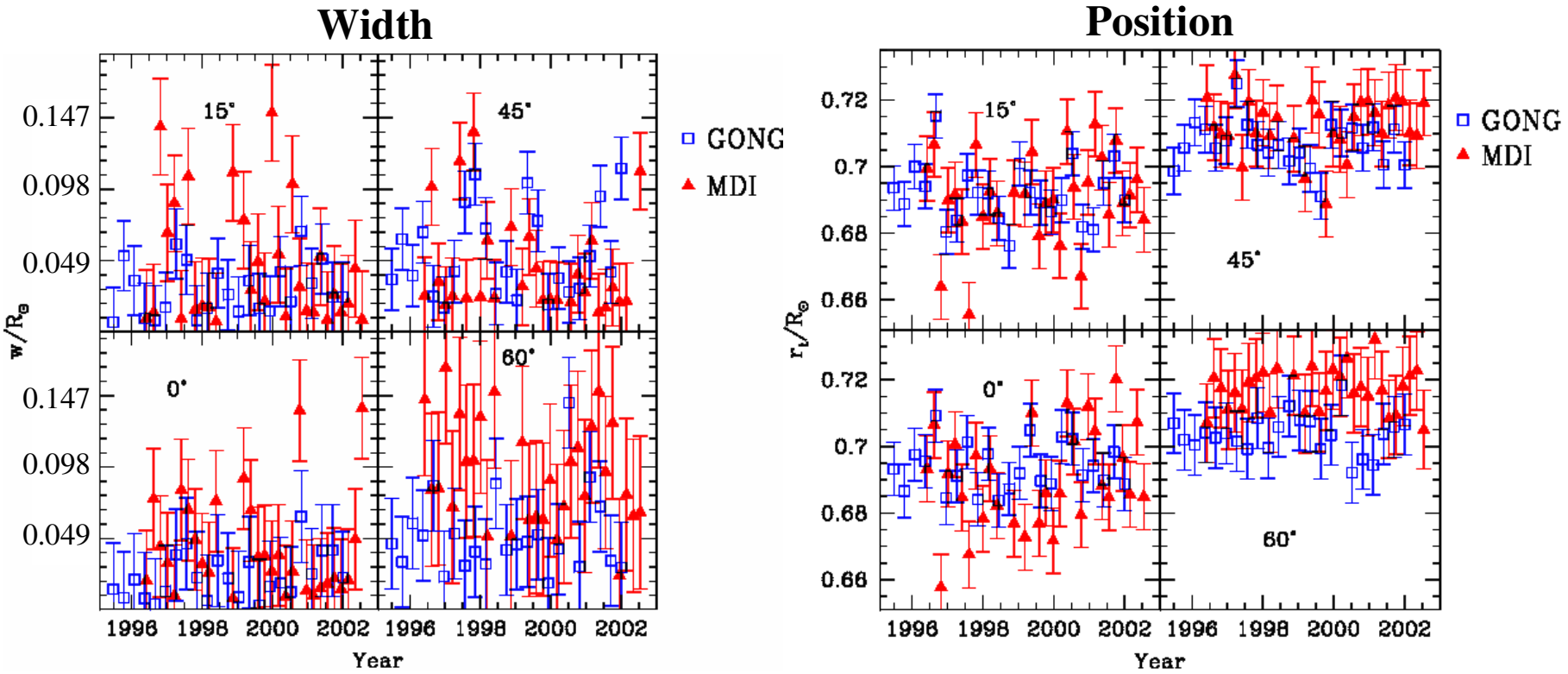
$r_t(60°)-r_t(\text{equator}) \sim 0.02R_\odot$

Similar Results from LOWL

(Charbonneau et al. 1999, Corbard et al. 2001)

=> The observed increase in thickness is only marginally significant

Results (3): Temporal variations, position & width



Courtesy S. Basu

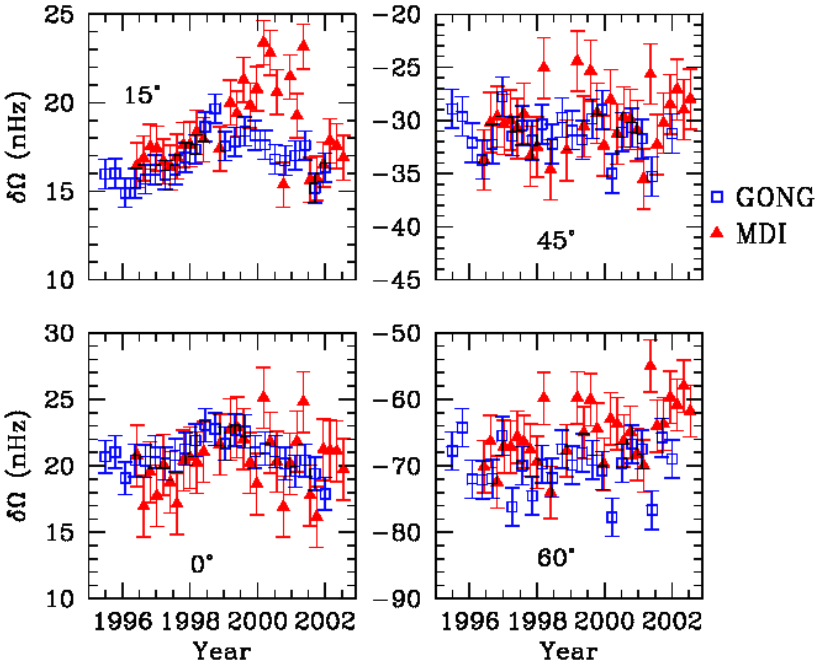
=> No systematic temporal variation detected for the width or position of the tachocline

Similar conclusions were obtained from 6 years of LOWL data

(Corbard et al. 2001, See poster from Eff-Darwich et al.)

Controversy (1): Temporal variations, 1.3 year oscillations?

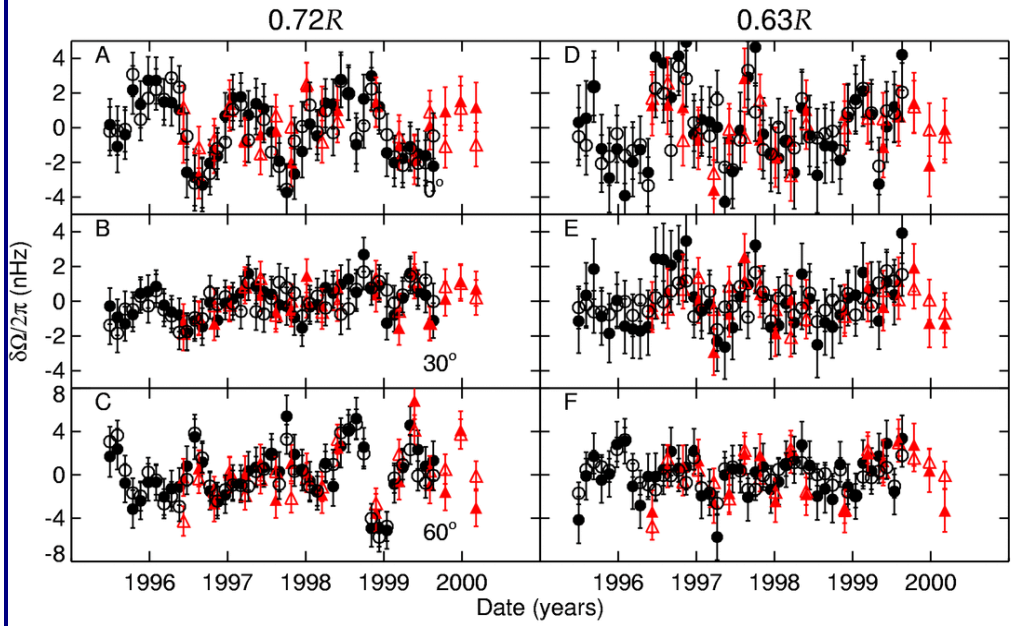
Jump across the tachocline



Courtesy S. Basu

- No significant variation in the jump $\delta\Omega$
- Some variation found in the spherically symmetric component of the jump. (Period 1.3 year not regarded as significant)
- Variation anticorrelated with Ω_c
- No significant variation found with MDI

Residual at the border of the tachocline



Howe et al. 2000

- Equator: period 1.3 year anticorrelated at $0.63R_\odot$ and $0.72R_\odot$. Regarded as significant. Amplitude ~6nHz
- 60° period ~ 1year less significant
- Found with both OLA and RLS methods
- Variations slightly less significant with MDI alone
- Seems to fade at solar maximum -> Transient oscillations(?) *See poster Toomre et al.*

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Controversy (2): Temporal variations, discussion

(1) *We are **not** exactly looking at the same thing in the two cases and the methods are different.*

-> Basu & Antia: look directly at the tachocline parameters using a 2D annealing method.

->Howe et al.: Use OLA and RLS which is a priori not appropriate for the tachocline.

$0.63R_\odot$ and $0.72R_\odot$ are indeed not in the tachocline at the equator ($[0.66\text{-}0.72]R_\odot$)

(2) *The variation at the boundaries are found in anti-phase as well as Ω_c and the symmetrical part of $\delta\Omega$*

=>Error correlation function ?

0.63 and $0.72R_\odot$ anticorrelated but low amplitude (0.2-0.4)

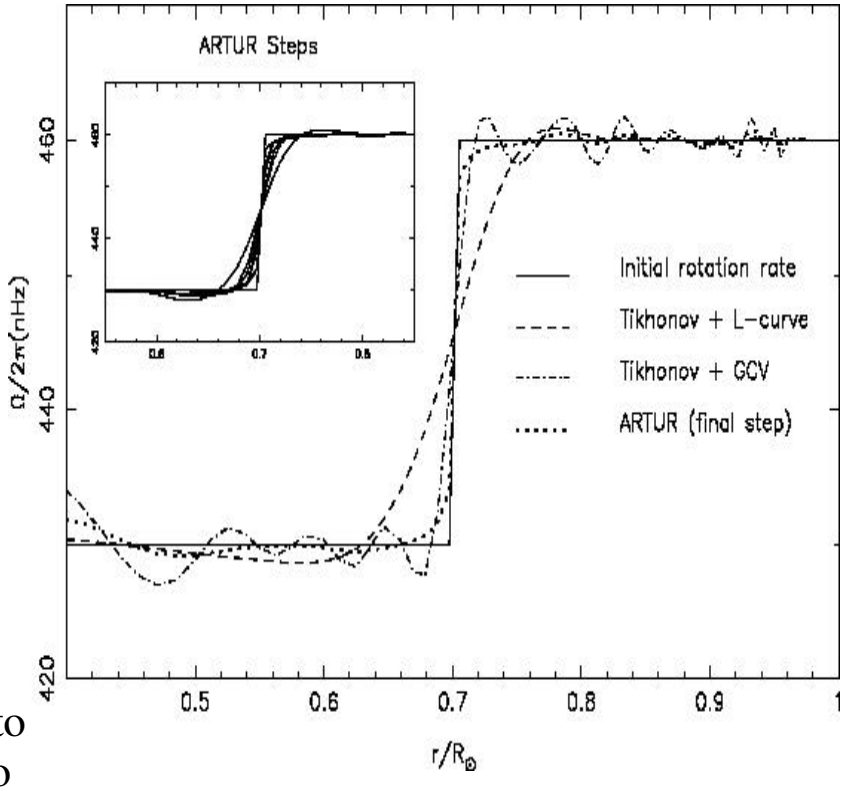
=>Varying properties of the noise ?

/ Choice of the regularization parameter?

=>Influence of $\Omega_c \neq \Omega_c(\theta)$ in 2D annealing ?

=>Gibbs phenomenon?

(Corbard et al. 99, Vorontsov et al. 2001)



(3) Results with adaptive regularization together with more intensive Monte Carlo Simulations could help us to address this important question. LOWL data should also be checked using time series < 1 year.

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THEORIES:

A BRIEF OVERVIEW OF RECENT DEVELOPPEMENTS

See also the recent reviews from
D. Gough
P. Gilman

Solar Physic 192 (2000)

Theories (1): Layers within the tachocline

key questions to determine the dynamic of the tachocline:

- Stratification / degree of subadiabaticity ?
  Greater subadiabaticity => greater negative buyoncy force => nearly 2D flow

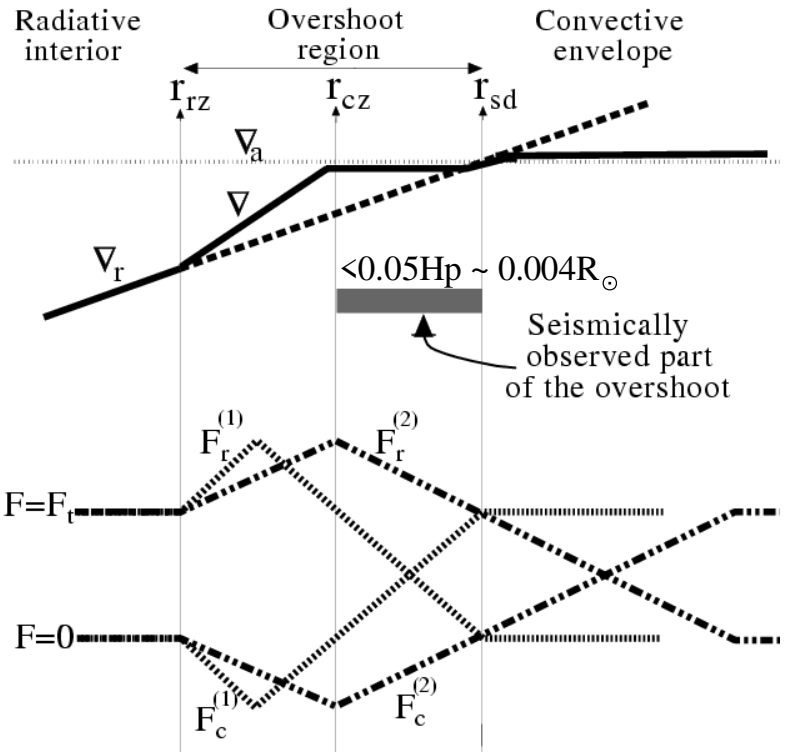
Rempel et al. 2000: Storage of a 10^5 G field on solar cycle time scale possible where $\nabla-\nabla_{ad}<10^{-3}$

Latitudinal component of the magnetic curvature force balanced by the Coriolis force due to a toroidal flow along field lines.

=>Significant variations of the differential rotation are expected during the cycle at the bottom of the CZ

- Statistical properties of convective fluctuations ?
  Xiong & Deng (2001) Full Non Local Mixing theory (low correlation turbulent velocities-Temperature)
  => larger subadiabaticity and smoother transition

Supported by recent numerical simulations:
Brummell & al. (2002) 3D Turbulent compressible Convection
Rempel & Rast (2002) 2D simulations



Corbard et al. 2001
Adapted from Zahn 91 & Canuto 97

Theories (2): Slow or Fast Tachocline ?

- Can the dynamo field penetrate the tachocline?
  penetration skin depth of an oscillatory field
  (Garaud 1999)

magnetic diffusivity

$$h = \sqrt{\frac{\eta}{2\pi\nu}} \qquad \nu = 22\ years^{-1}$$

=> Two class of exclusive models
(Gilman 2000, Forgács-Dajka & Petrovay 2002)

1)Fast tachocline (Lorentz time scale ~ 1 year)
$\eta > 10^9$ cm^2 s^{-1} Dynamic governed by the dynamo field
=> Focus on the CZ-tachocline interface

2)Slow tachocline (Diffusive time scale ~ 10^6 years)
$\eta < 10^8$ cm^2 s^{-1} Primordial field only
=>Focus on the Radiative interior - tachocline interface

Slow Tachocline (1)

First question: the thinness of the tachocline

Pb: $\Omega(\theta)$ -> ∇T + meridional circulation that tend to diffuse inward

Solution: Anisotropic transport of angular momentum

Spiegel & Zahn (1992) 2D hydrodynamic

Tachocline = Turbulent boundary layer created by shear instabilities in which horizontal viscosity is dominant

Problem: where the Turbulence come from?

Purely hydrodynamic Shear instabilities ?

Charbonneau, Dikpati, Gilman (1999) -> observed latitudinal shear linearly stable

Garaud (2001) -> weakly non-linear evolution of instabilities -> state of marginal stability

Dikpati & Gilman (2000) -> Shallow-Water Model : First approach of 3D case by allowing radial deformations of the layer -> Tachocline unstable to linear shear instability even in the purely hydrodynamic case

Slow Tachocline (2)

Second question: the rigid rotation of the core

Angular momentum transport by shear flow or internal gravity wave found not efficient enough

Solution:

An internal **magnetic field must exist**

MacGregor & Charbonneau (99) -> a very weak field ~10^{-3} G is enough would also
Rudiger & Kitchatinov (97) confine the tachocline to it's observed thickness

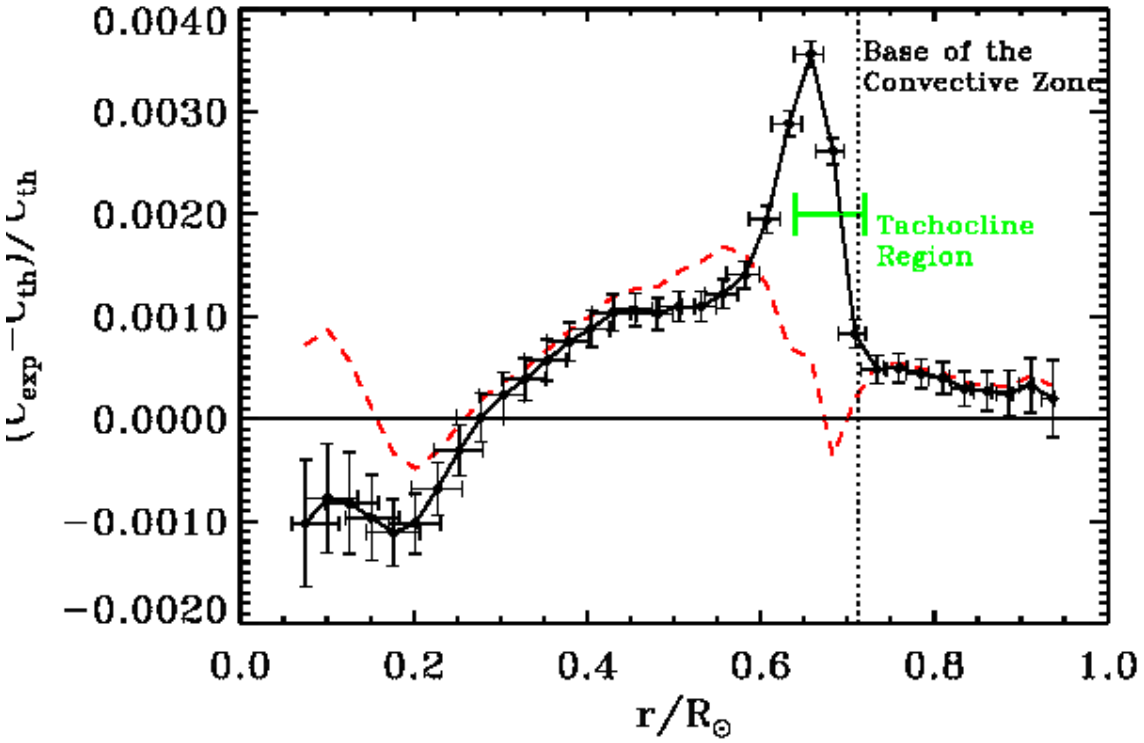
Problem: How to confine the magnetic field to the interior and avoid the inward spreading of the tachocline along the field lines ?

New Solution:

Gough & MacIntyre (1998) -> Balance between upward diffusion of the internal field and and downward advection of the tachocline laminar meridional circulation.

Garraud (2002) -> First numerical simulations show that at least qualitatively such a system is possible for a range of field strength, support a prolate tachocline and slow pole rotation.

Slow tachocline & Mixing of chemical elements



Time dependent macroscopic mixing below the convection zone.

=>**Remove the sound speed anomaly** if we relax Z/X or increase the pp cross section (within error bars)

=>**better agreement with surface ^{7}Li abundance**

=>**Maximum extent for the tachocline mixing ~ 0.05$R_\odot$** (based on ^{9}Be depletion)

Brun et al. 2002

Other possible mixing processes: meridional circulation, internal waves, magnetic field overshooting ***(Xiong & Deng 2002)***

Fast tachocline (1)

Dynamics gouverned by the dynamo genarated oscillatory magentic field

Magnetic Field as a source of anisotropic transport:

1) By direct action of Maxell Stresses

Forgács-Dajka & Petrovay(2002)-> An oscillating dipolar poloidal field of 10^3-10^4G is able to confine the tachocline to its observed thickness

Prolate tachocline varying on solar cycle timescale (prolatness < 0.02R)

2) Indirectly by generting turbulence via MHD instabilities

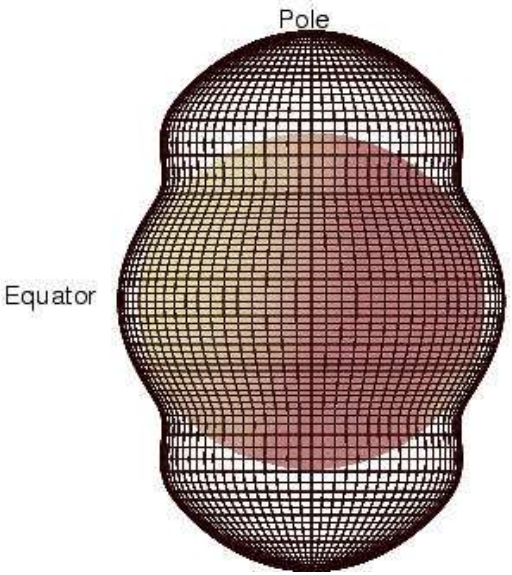
Gilman & Fox (1999), Dikpati & Gilman (1999), Gilman & Dikpati (2000,2002)

Model for the non linear developpement of the MHD instabilities:
Cally(2001), Cally,Dikpati,Gilman (2002)

Miesch(2001) --> coupled r/g inertial oscillations propagating towards equator (~1year)
(freely decaying turbulence, no imposed shear flow or magnetic field)

Fast tachocline (2) Prolateness and rotational jets in the tachocline

Gilman, Rempel, Dikpati (2002)



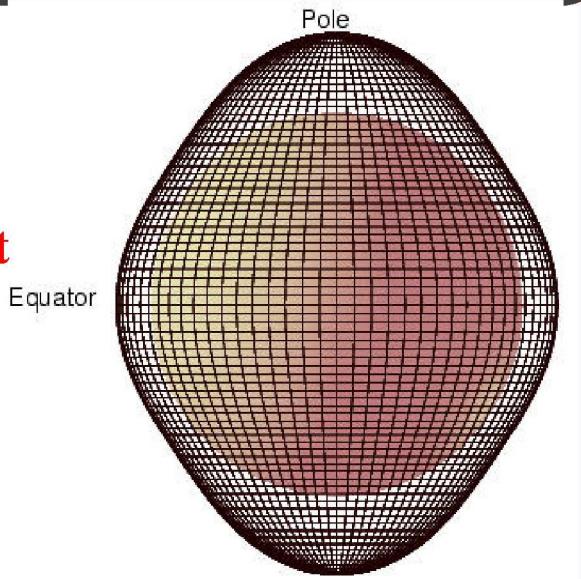
Prolateness ~0.02R =>

toroidal magentic field of

60-150KG in the overshoot part

or

600KG in the radiative part

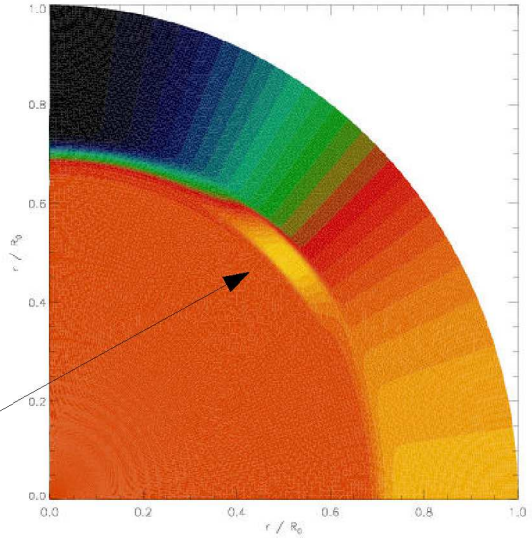


Strong toroidal field ring => Magnetic curvature stress

Can be balanced by:

1)Extra mass on the poleward side => prolateness
2)prograde rotational jet inside the toroidal ring
3)Both 1) & 2)

30°/ 40000Km band of 75000G at 45°



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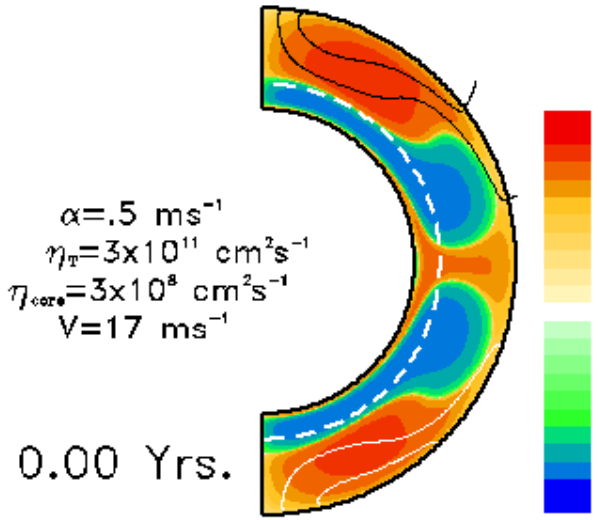
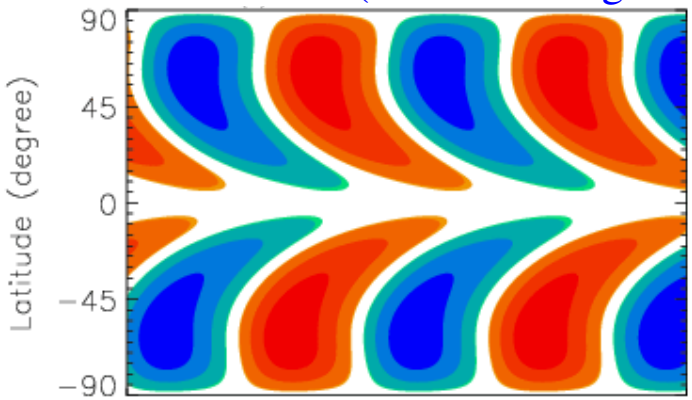
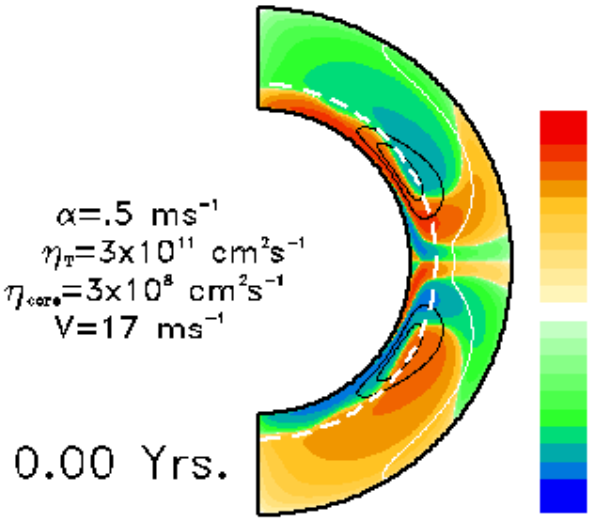
Tachocline α-effect & Magnetic Parity Selection
(Dikpati & Gilman 2001)



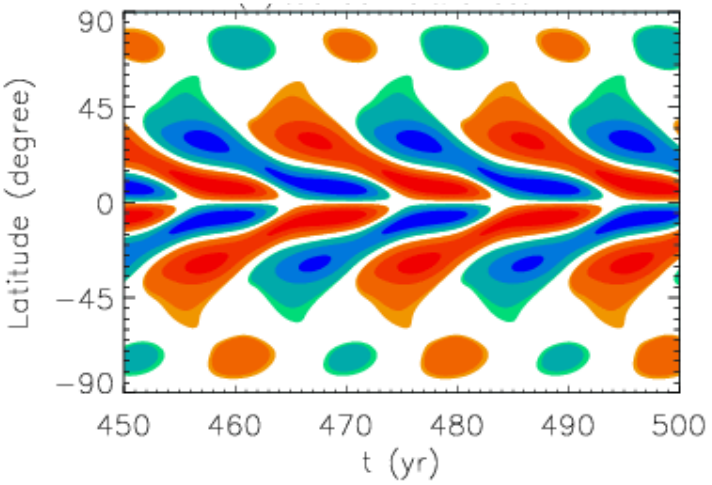
Surface α-effect (Babcock-Leighton)



Violates Hale's polarity Law



Tachocline α-effect from global instability of tachocline differential rotation



Select solar-like field symmetry

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