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Estimation of the spatial coherence outer scale for daytime observations

Seghouani N., Irbah A.

C.R.A.A.G - Observatoire d'Alger BP 63 Bouzaréah Alger Algérie

Borgnino J.

Université de Nice-Sophia Antipolis, UMR Astrophysique 6525, Parc Valrose F-06108 Nice Cedex 2 France

Abstract.
The theoretical expression of the transverse angular covariance of angle-of-arrival fluctuations of the incoming wave-fronts integrated all over the telescope pupil has been established in the case of one-layer model and using the Von Kàrmàn theory. This model depends on three atmospheric parameters: the spatial coherence outer scale $\mathcal{L}_0$, the Fried parameter r_0 and the size of the isoplanatic patch θ_0. We use this model to analyze the fluctuations observed on the solar limb, which finally consists on the estimation of the three parameters $\mathcal{L}_0$, r_0 and θ_0. The value of the spatial coherence outer scale $\mathcal{L}_0$ for day-time observations is not known. Its estimation is particularly important in the modelisation of atmospheric effects for solar observations with high angular resolution (solar diameter measurements at astrolabe for example). Two methods have been developed in other to estimate these parameters. They have been investigated and applied to solar images recorded at Calern Observatory Astrolabe (France). It is shown that the values obtained for $\mathcal{L}_0$ are smaller than those found for night observations. The obtained results are presented and discussed.

1. Introduction

The knowledge of the outer-scale of the atmospheric turbulence is of major interest in astronomy because it allows the implementation of high resolution techniques: speckle interferometry, long-baseline interferometry, adaptative optics. Its estimation using differents models, such as the Von Kàrmàn model, the Greenwood Tarazano model, the exponential model, in which this parameter is introduced to modelize the low frequency behavior of turbulence (Voitsekhovitch, 1995). In this study, the Von Kàrmàn model has been chosen since this model has proved to be in good accordance with observations (Davis et al, 1995). The outer scale L_0 depends on local conditions and particularly on altitude, it must be then differentiated from the spatial coherence outer-scale $\mathcal{L}_0$ which is an optical parameter related to the perturbed wavefront reaching the ground level (Borgnino 1990). An efficient method to estimate this outer scale consist in a statistical analysis of the angle-of-arrival (AA) fluctuations. Using a gaussian model, results have firstly been obtained by Consortini, Ronchi, & Stefanutti 1970 in the case of laser beam horizontal propagation. Then, Borgnino 1989 has proposed the use of these statistics to deduce $\mathcal{L}_0$ in the case of the observations of stars and Tallon 1989 has obtained first results. A more detailed theoretical study associated to numerical simulations has led to the proposal of a technique allowing the estimation of $\mathcal{L}_0$ from its effects on the variance and the covariance (Borgnino 1990; Borgnino, Martin, & Ziad 1992). Based on this principle, experimental estimation of the spatial coherence outer scale has been performed by Ziad et al 1994. Since, a Generalized Seeing Monitor (GSM) (Martin et al, 1994) has been built allowing the evaluation of astronomical sites: La Silla, Maydanak, Oukaimeden, Cerro Tololo, Cerro Paranal (Ziad et al 2000).

For daytime observations, the value of $\mathcal{L}_0$ is not known and its determination is of great interest in order to evaluate the optical effects induced by atmospheric turbulence on solar images, such as the ones recorded at Calern Observatory with the solar astrolabe (Laclare et al, 1996). In this paper, we present a new method to estimate $\mathcal{L}_0$ from a statistical analysis of AA fluctuations observed on the solar limb. We will use the concept of "equivalent impulse layer", theoretical turbulent layer situated at an altitude h and giving the same effects on the image that the whole turbulent terrestrial atmosphere. The theoretical one-layer model of the AA angular covariance is therefore developed. This model depends on three atmospheric parameters: the first one is the well-known Fried parameter r_0, the second one is the spatial coherence outer-scale $\mathcal{L}_0$ and the third one is the equivalent altitude h which allows us to determinate the isoplanatic angle θ_0. Two techniques have been developed allowing the simultaneous estimation of the three parameters r_0, $\mathcal{L}_0$ and h by confronting the model to the observations. These methods have then been applied to solar images recorded at Calern Observatory.

2. Theoretical one-layer model for the angle-of-arrival angular covariance

In order to establish the one-layer model for the AA angular covariance, we will use the expression of spatial covariance (Avila et al, 1997). Since the AA fluctuations α (averaged on each pupil) are observed in the direction perpendicular to the baseline, we will be interested only in the transverse covariance which, using the Von Kàrmàn model, is expressed as:

$$\begin{aligned} C_\alpha(\overrightarrow{B}) &= 0.0716\lambda^2 r_0^{-\frac{5}{3}} \int_0^{+\infty} df f^3 (f^2 + \frac{1}{\mathcal{L}_0{}^2})^{-\frac{11}{6}} \\ & \quad [J_0(2\pi f(B)) + J_2(2\pi f(B))][\frac{2J_1(\pi D f)}{\pi D f}]^2 \end{aligned} \tag{1}$$

This covariance has been established in the case of two pupils separated by a baseline $\overrightarrow{B}$, λ being the light wavelength, and D the pupil diameter. The wavefront inner scale l_0 is assumed to be equal to 0.

If we consider now, the AA fluctuations on two telescope pupils looking in two directions separated by $\overrightarrow{\theta}$ (figure 1) and assuming the case of one turbulent

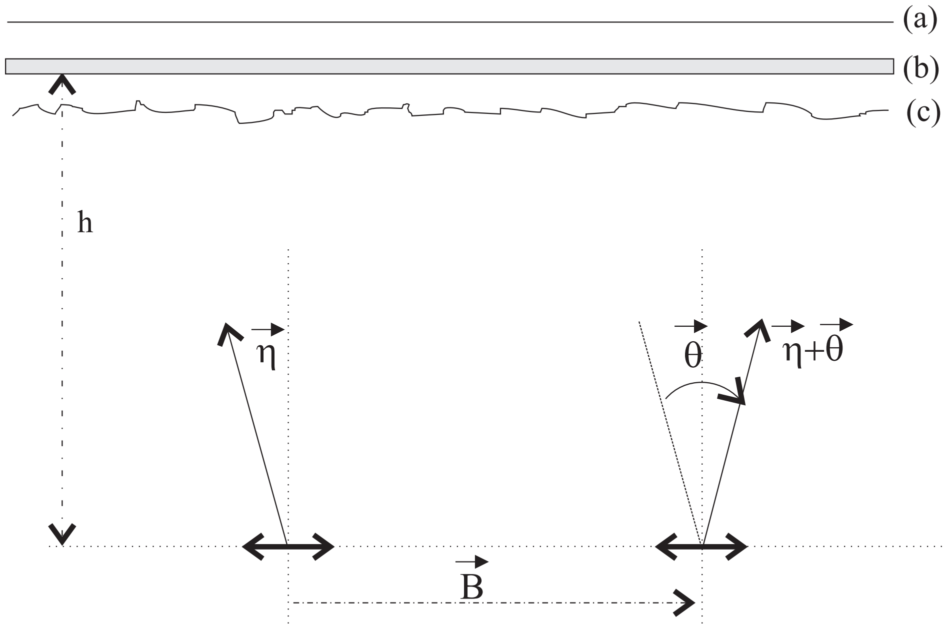

Figure 1. Principle and notations. (a) plane wavefront, (b) equivalent impulse layer, (c) perturbed wavefront.

layer situated at an altitude h, we can easily show (Bouzid et al, 2000) that the spatio-angular covariance can be related to the spatial covariance by:

$$\mathcal{C}_{\alpha,\overrightarrow{B}}(\theta) = C_{\alpha,\overrightarrow{B}}(\overrightarrow{\theta}h - \overrightarrow{B}) \qquad (2)$$

It is assumed that the angular direction $\overrightarrow{\theta}$ has been taken so that, at ground level, the spatial shift $\overrightarrow{\theta}h$ is oriented as the baseline $\overrightarrow{B}$. If we have a single pupil ($B = 0$) (case of solar astrolabe), The AA angular covariance is then given by:

$$\begin{aligned} \mathcal{C}_\alpha(\theta) &= 0.0716\lambda^2 r_0^{-\frac{5}{3}} \int_0^{+\infty} df f^3 (f^2 + \frac{1}{\mathcal{L}_0{}^2})^{-\frac{11}{6}} \\ & [J_0(2\pi f\theta h) + J_2(2\pi f\theta h)][\frac{2J_1(\pi D f)}{\pi D f}]^2 \qquad (3) \end{aligned}$$

This model established, we can elaborate inverse methods, in order to estimate the parameters r_0, $\mathcal{L}_0$, and h. It should be outlined, that h is the altitude of the equivalent layer which should give the same effects, observed at ground level, that the whole terrestrial atmosphere. The principle of these methods is presented in next section.

3. Estimation of the atmospheric parameters

With the model established above, we have defined a direct relation between the transverse covariance of the angle of arrival and the three parameters r_0, $\mathcal{L}_0$ and h:

$$\mathcal{C}_\alpha(\theta) = F_\theta(r_0, \mathcal{L}_0, h) \tag{4}$$

An experimental covariance $\mathcal{C}_{obs}$ can be obtained directly from solar edge fluctuations $\alpha(\overrightarrow{\eta})$ by:

$$\mathcal{C}_{obs}(\theta) =< \alpha(\overrightarrow{\eta})\alpha(\overrightarrow{\eta} + \overrightarrow{\theta}) > \tag{5}$$

A fitting of the model to the experimental covariance leads us to estimate the three parameters r_0, $\mathcal{L}_0$ and h. The isoplanatic angle θ_0, in the AA sense can be deduced by:

$$\theta_0 = \frac{r_0}{h} \tag{6}$$

To solve the above inverse problem two methods have been developed:

The least-square estimation: This method is the most direct to resolve the inverse problem mentioned above. The problem can be formulated in the following way:
What are the parameters ($\mathcal{L}_0$, r_0, h) which minimizes the cost function J ?

$$J = \|F_\theta(r_0, \mathcal{L}_0, h) - \mathcal{C}_{obs}\|^2 \tag{7}$$

As we are dealing with a non-linear model, the conjugate gradient algorithm (Fletcher & Reeves 1964)is an adequate way to resolve this minimization problem. However this problem is ill-conditioned, and the least-square solution diverges. In order to regularize this problem, we have imposed some physical constraints on the parameters:

$$\begin{array}{ccc} 0 < & r_0 & < r_{0max} \\ 0 < & \mathcal{L}_0 & < \mathcal{L}_{0max} \\ 0 < & h & < h_{max} \end{array}$$

The positivity constraint, especially, shows to be a very efficient way of regularization in this case (Seghouani 1995). The conjugate gradient algorithm is then used to resolve this constrained least-square minimization problem.

The abacus method: In order to estimate the atmospheric parameters without excessive processing time, one can use the following method.

The covariance is first integrated over two angular domains θ_1 and θ_2

$$I_\alpha(\theta_i) = \int_0^{\theta_i} d\theta \mathcal{C}_\alpha(\theta), \quad i = 1, 2 \tag{8}$$

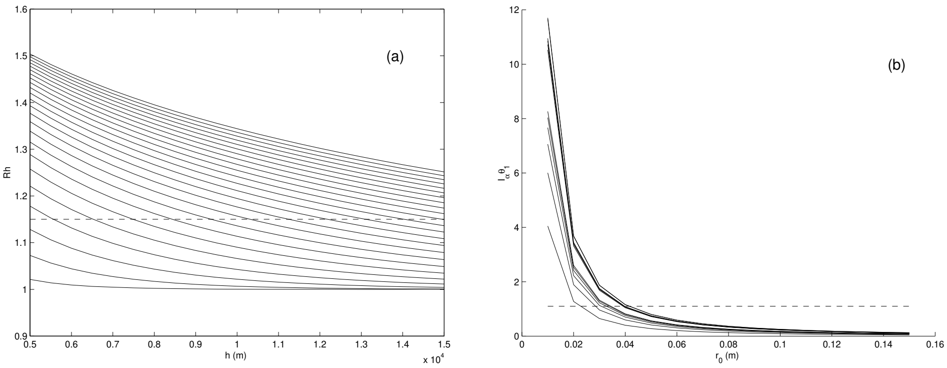

Figure 2. Abacus method: a) Abacus of the ratio R_h as a function of h for different values of $\mathcal{L}_0$ (from 1 m to 20 m) . The intersection with the experimental value of R_h gives the set of potential solutions ($\mathcal{L}_0$,h). b) Abacus of $I_\alpha(\theta_1)$ as a function of r_0 for a finite set of couples ($\mathcal{L}_0$,h). The intersection with the experimental value of $I_\alpha(\theta_1)$ gives the set of potential solutions($\mathcal{L}_0$,h,r_0).

Since $\mathcal{C}_\alpha(\theta)$ and thus $I_\alpha(\theta)$ are linear with regards to r_0, the ratio R_h defined below, depends only of the two parameters $\mathcal{L}_0$ and h:

$$R_h = \frac{I_\alpha(\theta_2)}{I_\alpha(\theta_1)} = R_h(\mathcal{L}_0, h) \tag{9}$$

With an a priori knowledge of the variation range of $\mathcal{L}_0$ and h, one can then construct an abacus of R_h as a function of $\mathcal{L}_0$ and h as shown in the figure 2(a).

On the other hand, the ratio R_h can be computed experimentally from observations. We can therefore deduce from the abacus a set of couples $\{(\mathcal{L}_0, h)\}$ that are potential solutions. For each possible solution ($\mathcal{L}_0$, h) we must now determinate the associated Fried parameter r_0. For this purpose, we compute $I_\alpha(\theta_1)$ as a function of r_0, for each potential solution ($\mathcal{L}_0$, h):

$$I_\alpha(\theta_1) = \int_0^{\theta_1} d\theta \mathcal{C}_\alpha(\theta) = f(r_0), \tag{10}$$

This operation leads us to the construction of a new abacus as shown in figure 2(b).

The observations give us an experimental value for $I_\alpha(\theta_1)$ and thus a new finite set$\{(\ r_0,\ \mathcal{L}_0,\ h)\}$ of potential solutions. We can choose the best one by means of the least-square estimator.

4. Application to solar astrolabe images recorded at Calern Observatory

The previous methods have been applied to images recorded at Calern Observatory (Observatoire de la Côte d'Azur, France) with the solar astrolabe. Indeed,

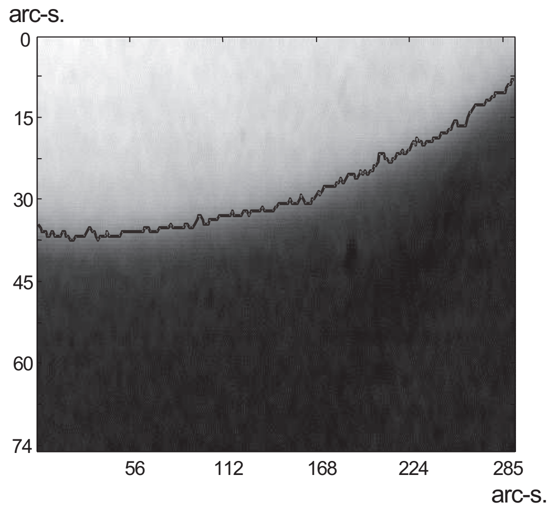

Figure 3. Image recorded with solar astrolabe and edge extraction.

the turbulent atmosphere is responsible of the fluctuations observed at the solar limb. A statistical analysis of these fluctuations can therefore leads us to qualify the atmospheric effects on the recorded images.

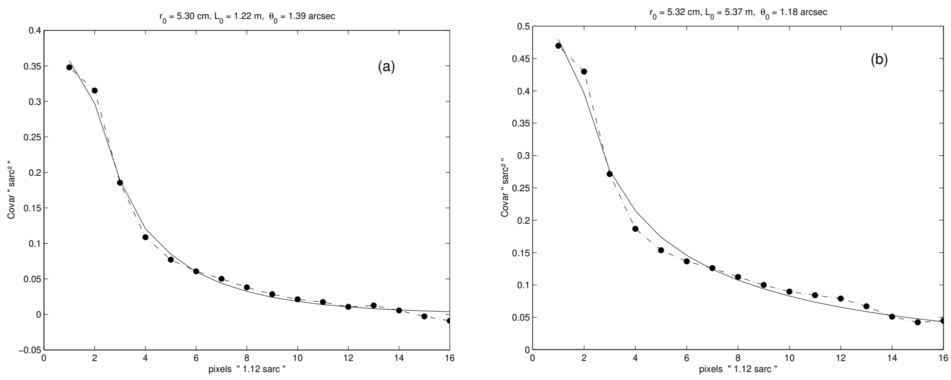

Figure 4. Examples of two experimental covariances fitted by the model.

The solar astrolabe gives direct and reflected images; only the direct images will be used in our case. The instrument has an elliptic objective with major axis of 8 cm and the minor one of 5 cm. The images of solar edge are recorded on a CCD camera with resolution of 1.12 by 0.74 arcsecond. The images are recorded at a cadence of 2 images per seconde, with an integration time of 20 ms. The images are then 'cleaned' by means of a wavelet technique in order to remove all impurities (Irbah et al 1999). After edge detection (figure 3), the fluctuations are extracted in the perpendicular direction of the telescope pupil using a parabolic approximation. The fluctuations are observed on 285 arcsecond angular field. After the suppression of the residual noise, taking into account the CCD cut-off frequency (Irbah et al 1993), an experimental covariance is computed. Each experimental covariance is computed using a set of 50 images. The covariances are computed on 128 points (143 arcsecond) but only the first 16 points (18 arcsecond) are used to estimate the parameters. Examples of experimental covariances fitted by the model after parameter estimation are presented in figure 4. The table below shows some parameter estimations obtained from images recorded during the same day at Calern Observatory in June 1990.

Table 1. Some results obtained during one day of observations at Calern Observatory from the solar astrolabe images.

r_0 (cm)	$\mathcal{L}_0(m)$	h (m)	θ_0 (arcsec)
2.40	2.92	8594	0.58
2.05	1.37	5300	0.80
5.00	1.22	7437	1.39
5.26	2.88	8415	1.29
5.58	1.93	8173	1.41
5.30	2.30	7822	1.40
5.63	4.08	8095	1.43
5.38	3.87	8576	1.30
4.33	3.12	9280	0.96
5.32	5.37	9261	1.18
5.22	3.99	9338	1.15
5.30	1.05	7733	1.41
3.87	0.57	6850	1.16

The above results show that we obtained small values of $\mathcal{L}_0$, namely between 2 and 5 meters, while nighttime values of $\mathcal{L}_0$, with GSM experiment (Ziad et al 2000; Martin et al 2000) vary around a typical value of 25 m.

5. Conclusion

In this paper, a new expression for the AA angular covariance has been established. It is based on the principle of 'equivalent impulse layer', the Von Kàrmàn theory and depends on three atmospheric parameters that are the Fried parameter r_0, the spatial coherence outer scale $\mathcal{L}_0$ and the equivalent altitude h which leads to the isoplanatic angle θ_0. Two inversion methods allowing the simultaneous estimation of the three parameters have been developed. They have been applied to solar images recorded at Calern Observatory. First results for $\mathcal{L}_0$ have been obtained. They show that the obtained values for the outer scale are smaller than the ones obtained for night-time observations. These preliminary results need to be confirmed by long-time observations which will allow a temporal evolution analysis of the atmospheric parameters.

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